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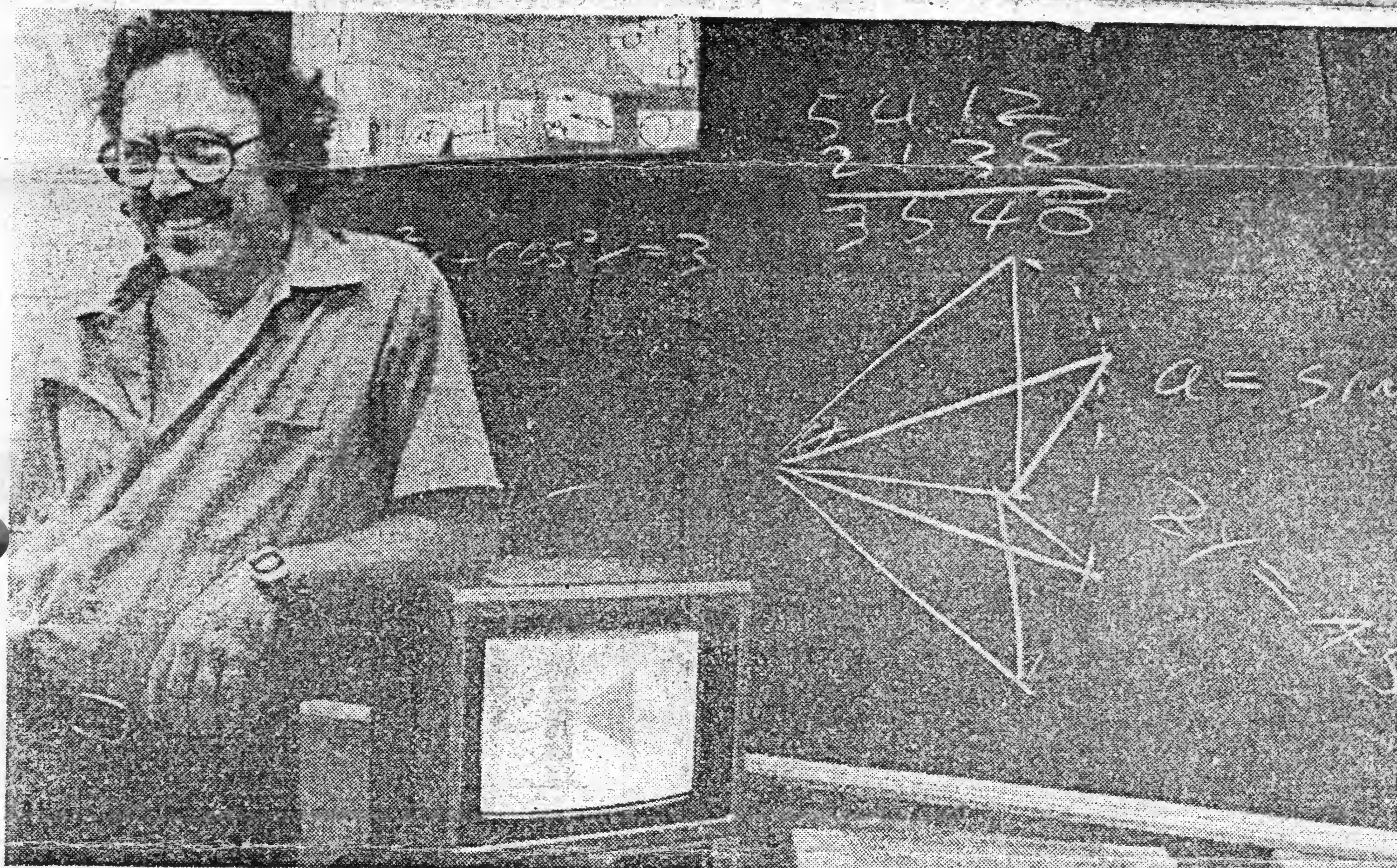


Philadelphia Inquirer / VICKI VALERIO

Klotz, at a terminal used in his video program for trig, says the project will be available to schools in a year

Fun and games

Trigonometry going the way of Asteroids

By Dick Pothier
Inquirer Staff Writer

Students are pouring millions of quarters into video-arcade games, such as Space Invaders and Asteroids, and when they're not doing that, a lot are sitting at home playing video games on their TV sets.

All of that video activity gave Swarthmore College math Professor Eugene A. Klotz an idea — an idea that might change the way students learn trigonometry, long considered among the toughest courses offered in high school and college.

Now, using a bank of inexpensive Atari home computers — chosen because their performance and price are suitable for the project — and a $150,000 grant from the National Science Foundation, Klotz and a team of educators and computer people are designing game-like video programs for teaching trigonometry.

To know that trigonometry deals with the ratios between the sides of a right triangle and either acute angle, the relations between those angles and the use of those ratios to find the unknown sides of angles or triangles is to understand, in part, why the subject is considered tough.

"Trig," as it's not so fondly called by millions of students and former students, "is generally at the top of most people's lists of disasters in high school," Klotz said in an interview Tuesday.

In addition to standard keyboard controls, the video-game design will employ joy sticks,

(See TRIG on 16-A)

Trig going the way of Asteroids

TRIG, from 1-A

movable controls that users twist and jiggle to make things happen on the screen. Such controls are used with most games that attach to home T-V sets.

Klotz and his team are developing colorful, animated, geometric displays of circles, triangles and trigonometric functions. Students will be able to maneuver the displays with the joy sticks.

Control of the displays will allow the students the freedom to regulate the speed and content of a trig program, thus controlling the rate at which they learn on an individual basis.

"We plan to have spectacular color displays and graphics — and if the student didn't get it the first time, he or she will be able to back up and try it again," Klotz said.

"The use of color and animation will clarify complicated figures, allow added emphasis in figures and text and contribute significantly to visual interest. And we think an entertaining, visual, video-game approach will help get rid of 'math anxiety,' a common problem today," he said, referring to an excessive fear of the perceived difficulty of learning mathematical concepts.

Klotz said trigonometry proves to be a major stumbling block for many high school and college students. Trig functions, such as sine, cosine and tangent, he said, are more easily visualized than talked about.

Klotz and his team of teachers and programmers are designing a trig-teaching program that is fun, allows students to "browse" through the computer program as easily as they can through a book and includes lots of color, motion and perhaps even graphic road-map signs to help them.

"We could even have a truck pushing a line . . . things like that," he said.

Once the programs are perfected, probably in 10 months to a year, Klotz said, they will be available for any high school, college or other teaching institution that needs them. They are not being designed for home use.

Besides its inherent difficulty as the first "entry into higher-level thinking for many high school students," trigonometry also presents particular difficulty for some groups of people, Klotz said.

"Minorities and inner-city kids, for example, who may be less verbal in standard English and more visual, are very likely to be familiar with video-arcade games and would find control of this equipment familiar and comfortable for them. And women often have a lot of 'math anxiety' because a lot of girls are pushed into believing that math is one of the areas not traditional or appropriate for women to study," he said.

In addition, he said, the handicapped might find the simple, joy-stick control easier to use to learn trig than textbooks and pencils, and the programs would also be useful for refresher and remedial work in colleges

The programs the Swarthmore team is developing will be usable on Atari home computers that cost as little as $400, Klotz said.

One of the program's top consultants is Ted Nelson of Swarthmore, author of *Computer Lib* and a number of other publications that advocate more "creative computing."

"I'm trying to put in things like angles that creak and groan when they open and close," Nelson said. "I've always found that kids love computer play — it's only adults who are afraid of computers."

(For the Computers in Mathematical Sciences Education Newsletter, 10/28/81)

<u>The Swarthmore Computer Trigonometry Project</u>

Eugene Klotz

Department of Mathematics

Swarthmore College

Swarthmore PA 19081

As recipients of a grant under the NSF-NIE Program for Mathematics Education Using Information Technology, we are developing a series of trigonometry tutorials. We are using Atari computers (discussed further below) and will require a disk drive and 48K memory (we may be able to squeeze down to 32K).

<u>Approach:</u> Our work is visually oriented, using color graphics as much as possible, mimimizing text. The pace and order of presentation are under user control: user imput is via joystick and the four special keys on the right of the Atari keyboard (the only keys functional throughout the programs). Upon power-up (or System Reset) the user is given the option of an instruction set to make him/her comfortable with the joystick and the various features of the programs. Whenever possible, we use the joystick to elicit motor involvement on the part of the student.

To assure educational soundness and relevance, our units are being written by a team of three high school mathematics teachers. For motivation we use color (also important for clarity and for emphasis), sound (earphones can, of course, be used in quiet environments), and games (all with learning objectives). Our units are aimed at the mainstream high school trigonometry population, and their stated intention is as supplemental material. However, we expect the units to be useful to a much broader population.

<u>Hardware:</u> We are using Atari computers because of their low price and many graphics features. These include a number of graphics modes of varying color capabilities

and resolutions (highest: 190 x 320). In addition, there are several character modes which can be used for letters of different sizes and colors, and are also available for user redefinition, giving further possibilities for both graphics and for animation. Moreover, there is a flexible display list system which allows mixing many graphics and/or character modes on the same screen, and also greatly increases color possibilities within the same mode. The "player-missile" graphics system provides several vertical strips within which one can place small objects - in colors independent from the screen colors - and then conveniently move the objects, if desired. Fine scrolling is a very important hardware feature for us. It allows one to treat the screen as a window into a much larger world, by moving smoothly in any direction. This can afford a good solution to the very serious and universal problem of how to display more information than conveniently fits on the screen at one time.

Some of the Atari graphics features require sophisticated programming and can take some time to master. Fortunately, the Atari organization can be very helpful. Several of the persons in their Software Department Support Group have commendable attitudes toward human engineering, and useful knowledge thereof.

Software Development Environment: Our underlying programming is being done in the FORTH language. Although this applications language has been used for everything from pocket language calculators to radio telescopes to arcade games, it is little known in the academic community. The reasons for our choice were: speed, compactness, flexibility, total control of computer, ease of programming, and availability. We are quite pleased with FORTH, but should point out that it is different from most standard programming languages. Fortunately, a splendid new book on FORTH has just been published (Starting FORTH by Leo Brodie, Prentice-Hall).

Timetable: We plan to test some of our preliminary units in several local high schools during the second semester of this academic year. By the end of next summer (1982), we expect to have completed an Introductory Trigonometry Package, which will cover angle measure and the trigonometric functions. This package will be sent to MicroSIFT and to CONDUIT for comments, evaluation, and possible distribution. Commercial vendors may also be approached. We hope the Introductory Package will be available to users by the second semester of the following academic year (spring, 1983).

We shall be looking for further funding in hopes of producing a package covering: preliminaries (special right triangles, pi, etc.), triangle trigonometry, trig identities, inverse functions, and polar coordinates. We are laying aside the work we have done in these areas in order to focus on the Introductory Package.

E.Klotz, Nov.1, 1982

THE SWARTHMORE TRIGONOMETRY PROJECT

WHERE WE ARE

Our goal is to produce a set of disks which will cover all of the major aspects of trigonometry:

- angle measure:
  - degree measure
  - radian measure (including applications to arc length)
- the trigonometric functions
- triangle trigonometry
- trigonometric identities
- inverse trigonometric functions
- polar coordinates.

Thus far, we have essentially completed the material on angle measure and on the sine function, in addition to background material (instructions, TV calibration, credits).

We are actually much further along than this would indicate. We have written, programmed, revised, and re-revised the material we now have in developing a viable and pedagogically sound approach. Moreover, we have developed literally hundreds of subroutines, which give us complete control of the graphics and audio features of the Atari. (We even have programs to write programs to construct special display lists and the like). We have an efficient production routine, a number of highly trained programmers, and three high school teachers who have become experienced in writing software.

If we are able to obtain further funding in the near future, we anticipate being able to finish the entire project by the end of the summer of 1984. We could hire one of our teachers half time next semester, and we have one programmer (who worked for us for two summers) that we could hire full time during the academic year. The remainder of the work would be concentrated in the two summers.

There follows a list of units which are currently ready or nearly ready.

TRIGONOMETRY UNITS, November, 1982

BACKGROUND:

Instructions (on Sample Disk I)
TV Calibration (Disk I)
Credits (Disk I)

ANGLE MEASURE:

Degrees (Disk I)
Angles and Rotations (Disk I)
Degree Estimation Game

Why Radians?
Radians: another way of measuring angles (on Disk II)
Exploring Radians
Radian Estimation Game (Disk II)
Radians and Pi

Radians vs. degrees (Activity)
Degree to radian conversion (Tutorial/Activity)
Radian to degree conversion (Tutorial/Activity)

Radian measure and arc length (Disk II)
Arc length estimation game
Arc length problems

Wrapping Real Numbers

The Radian Game

THE SINE:

History of "Sine" (on Disk III)
Sines of angles in right triangles (Disk III)
Sines of angles in a coordinate system
Sines and Coordinates- Exploration (Disk III)
Sines between 0 and 360 degrees (Activity)
Sines of arbitrary angles
Sines of special angles (Tutorial/Activity) (Disk III)
Exploring sines
Sine Estimation Game
Plotting the sines of special angles
Graph of the sine (Disk III)

KINDS OF UNITS

We have evolved the technique of breaking up the material into four kinds of units:

Tutorial, in which basic concepts are presented and illustrated.

Activity/Exploration, which give the user tools and situations for understanding and "getting the feel" of the basic ideas. Worksheets are provided for this type of unit in order to suggest directions and provide focus.

Game, which test the student's mastery of a concept, and give practice in estimating numerical quantities. Normally, games can be played on a number of levels. A minimum level of competency is suggested.

Overview/Perspective/Historical, usually short, straight text. These are used when the situation warrants it.

The types of units are interspersed as pedagogy demands.

HOW STANDARD EVALUATIVE CRITERIA HAVE BEEN MET

There follows a list of criteria normally used to evaluate educational software, together with an assessment of how we meet (or fail to meet) these criteria:

Ease of use: Through class testing of students and trying out our materials on naive college and high school teachers, we have found that our Instruction unit thoroughly explains everything which the user needs to know, and does so in a pleasant way. All user input is via joystick, together with the three special keys to the right of the Atari keyboard. Those who have never used a joystick in playing an arcade game are given instructions as to how to hold the joystick, and adequate opportunity to become familiar with its use.

In addition, we shall provide a booklet, "User Information", which should allow anyone to use our disks, whether or not they have ever previously used or turned on a computer.

Educational Soundness: This is being assured by having the units written by three practicing high school mathematics teachers, in collaboration with the project director, a professor of mathematics with experience in teaching remedial and pre-calculus mathematics. In addition, we have worked in consultation with a member of the Swarthmore College Department of Education.

Use of Computer Capabilities: We have used the full graphics, sound, and computational capabilities of the microcomputer with the most advanced graphics and sound features. We have taken care to use these features to assist in the educational process, and not detract from it.

Our programming techniques and approaches, and even our programming language (FORTH) are the most advanced currently available.

Student Control: The student has complete control over the order and the pace at which he/she proceeds, and always has the option of going back over previous material.

Branching: Early approaches to computer aided instruction relied heavily on the computer as a device for routing the student, based on previous responses. Our own approach, as mentioned directly above, is to give the student complete control. For example, if the student seems to be having difficulties in a game, we suggest that a

level be repeated or that a previous unit be reviewed, but we do not force either action.

Paraphrasing Dorothy Parker, we believe that you can lead a student to learning but you can't make him think.

Documentation for User and Teacher: In the "User Instruction" booklet, alluded to above, we shall provide:

instructions for first-time computer users
a troubleshooting checklist
suggestions for the home user
suggestions for the teacher
a table of contents to the units (each disk will contain its own table of contents, as well)
a discussion of the various types of units
a list of the objectives of the various units (these will also appear with their units on the disks)
worksheets for all units which have exploration or activity possibilities (a summary of the worksheet's suggestions will also appear on disk with the unit).

Clear Objectives: Each unit has written objectives which are available on the computer both before and after the unit. These objectives will also be collected in the User Information booklet.

If the reasons for studying a topic are not readily apparent, as is the case for radian measure, there is a discursive unit outlining the purpose of the topic.

Information Increments: Our testing has shown that the units present information in increments appropriate to the user. We avoid overload and clutter through such techniques as user-controllable pace, the option to review previous material, and color coding.

Glitch Free: We look carefully for bugs, glitches, and spelling errors, and are maintaining especially high standards.

Stereotype Free: We have made considerable effort to assure that these units do not suffer from ethnic, socio-economic, or sexual stereotyping.

Appropriate Medium: Throughout, we have searched for educational approaches which are most suitable for the computer. For example, we have been careful to avoid unnecessary text, and what text we do use is presented in ways particularly appropriate to the computer (smooth scrolling or reversible sequential replacement).

We present ideas using graphics techniques which a teacher could not duplicate on a blackboard. While finer graphics are available via photographic and video technologies, these approaches cannot match the computer's interactive capabilities.

SWARTHMORE COLLEGE
SWARTHMORE, PENNSYLVANIA 19081

DEPARTMENT OF MATHEMATICS

(215) 447-7246

May 22, 1981

Mr. Robert Kahn
1196 Borregas Avenue
Sunnyvale, CA 94086

Dear Mr. Kahn:

I have recently obtained a National Science Foundation Grant to do color graphics on the Atari. Enclosed is a copy of my grant proposal. I hope to use the FORTH computer language and have discovered that your Apex version is not yet ready for release. I'd very much appreciate a copy of your inhouse FORTH, and would be willing to sign any necessary papers in order to obtain this.

Moreover I would find that it would be useful to have FORTH "words" dealing with graphics, scrolling, and input-output functions (including joystick control). If these words were available, I would again be quite willing to sign any necessary agreements of confidentiality.

With thanks in advance for your help,

Sincerely,

Gene Klotz

Dr. Eugene A. Klotz
Professor of Mathematics

EAK/g

EDUCATIONAL GAME PROPOSAL

Swarthmore Trigonometry Project
Eugene Klotz
4/28/83

Goals: To develop one or two educational games aimed at the high school algebra and trigonometry level (scenarios are appended). All are suitable for both home and school, and should run on an Atari 400.

Duration: June 15-Sept. 1, 1983.

Cost: $25,000

Product: The choice is open for discussion. We would produce in final form one of the following: (1) both action games (scenarios follow), or (2) the adventure game and at least half an action game, or (3) the quiz ("jeopardy") game and at least half an action game, or (4) the activities package.

Continuation: Subject to further negotiation, the project could continue past Sept. 1 to develop other games or (in the second and third options above) to finish games already begun.

Personnel: Our two teachers, three recent college graduates, and project director have all had two years experience working together on Ataris. One or two junior programmers may be integrated into the project.

Important Time Constraint: This proposal is only feasible if we can come to an agreement by May 15, after which the personnel must begin making other plans.

(1)Frog and Fly (or sheepdog and sheep--the actual context may change as the details of the game are worked out.)

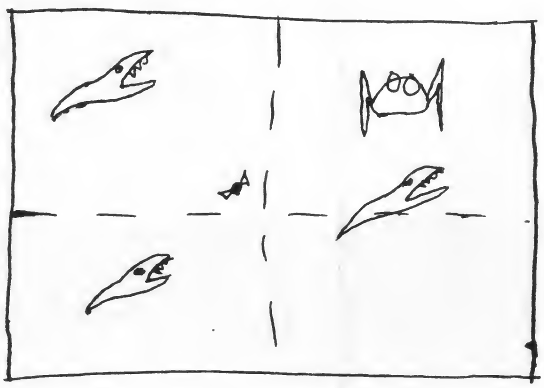
Objective: to develop proficiency at short-answer questions involving the measure of angles and the value of trigonometric functions of special angles.

The player uses the joystick to control the motion of the frog. The frog tries to catch the fly, while avoiding alligators and other obstacles.

In the beginning, the fly is buzzing about in the center of the screen. A message comes on the screen just before the fly takes off, giving a hint to the direction of his flight, such as:

"Launching point is at 50°" (or at $\frac{2\pi}{3}$ radians, or at θ, where $\sin\theta = 1$, or at θ where $\sin\theta = \frac{\sqrt{3}}{2}$, etc.)

This gives the player time to discover where the fly will be. The fly reaches the launching point and travels outward along a radius toward the edge of the screen. The frog must reach the fly before it gets to the edge of the screen. The player gets more points for catching the fly closer to the center, since this indicates that the player made a good prediction of the fly's starting point.

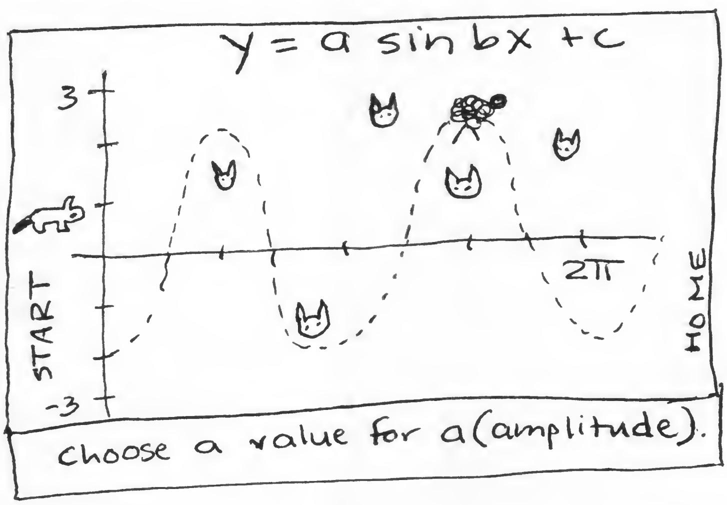


(2) Black Sheep

Objective: to develop familiarity with the equations of sine functions and to learn how to change the amplitude, frequency, and y-intercept.

An introduction sets up the situation of the sheep, lost among the wolves, and of the dog who must guide it home.

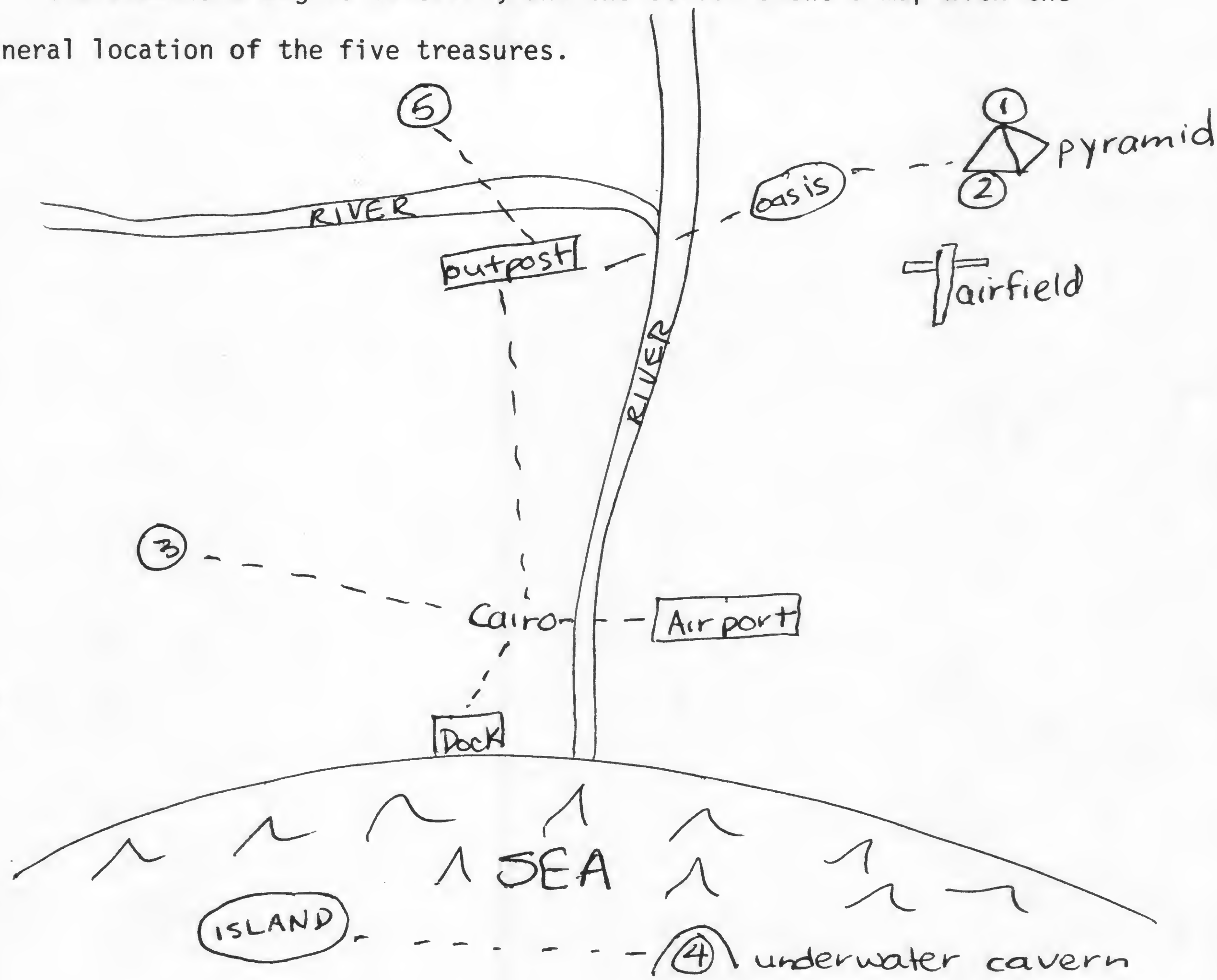
The player chooses values for a, b, and c (earlier versions might involve choosing just one or two of these) according to the instructions in the text window. The dog then follows the graph of this equation across the screen. The player tries to choose an equation which will cause the dog to intercept the lost sheep while avoiding the wolves.



The Pharaoh's Five Fortunes

Objective: to present traditional trigonometry word problems in context and to provide motivation for solving these problems.

The adventure begins in Cairo, and the screen shows a map with the general location of the five treasures.



The player must first purchase equipment, camels, maps, etc. at the Bazaar. Prices of the items can only be determined by solving problems put forth by the wily Arabs. For example: A trig table costs 7 shekels more than a compass. Two trig tables and four compasses costs 74 shekels. You want to buy one trig table and a compass. How much will you pay?

The player then sets out in search of one of the fortunes. Each fortune can be found by solving word problems, but each requires a different kind of problem, so that students who know some trigonometry can find one or two fortunes; as they learn more, they can continue to play the game to find the other fortunes.

Fortune #1, for example, might involve solving right triangles. You rent an airplane to fly to the airfield near the pyramid. The airfield is 80 miles from the Cairo airport. You must fly at an altitude of 3000 ft., and your plane must descend at an angle of 4° or less. How far from the airfield must you begin your descent? (A diagram depicting this situation can be purchased at this point, in order to aid the student in solving the problem.)

The height of the pyramid can be found by solving a similar problem, and eventually fortune #1 is found. At this point, the player can retire with one treasure, continue on to other treasures, or save the current position and continue play at a later time.

Fortune #2 requires that the player know the values of trig functions in all four quadrants. The screen shows the base of the pyramid, marked off in quadrants. The cursor indicates the player's present position. The player finds out that the treasure is at an angle of 225°, but doesn't know which axis is the positive x-axis. By directing the joystick, the player moves into each quadrant and finds hints as to which quadrant it is--for example: the sine is negative in this quadrant, and the tangent is positive. This must then be quadrant III, and eventually the second treasure is found here.

The player can end the game at any time, come back later to continue, or begin the game again and make different decisions or follow a different route.

"Jeopardy"

Objective: to provide motivation for solving trigonometry problems of varying kinds and difficulty.

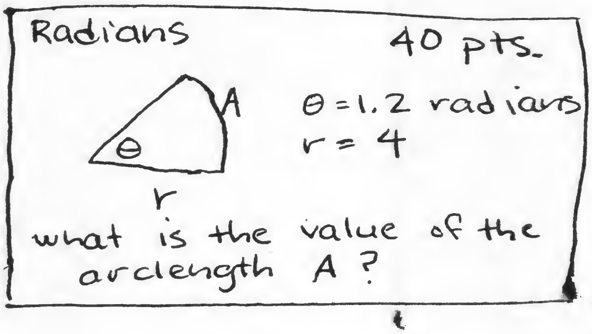
Many teachers play a version of this game with their entire class. The advantages of a computer version are that a few students (up to 4 at a computer) can play while the teacher is busy with the rest, graphical depictions of certain problems can be shown quickly on the screen, and randomization of certain variables allows the computer to generate an enormous number of problems.

The game begins with a chart which shows the categories and the point value of the questions. The text window tells whose turn it is to choose a question. That player then positions the cursor by using the joystick. The question appears on the screen, with a diagram if appropriate, and the first person to push the red button on the joystick puts in an answer. If correct, the point value is added to that player's score. If incorrect, another player gets the opportunity to answer.

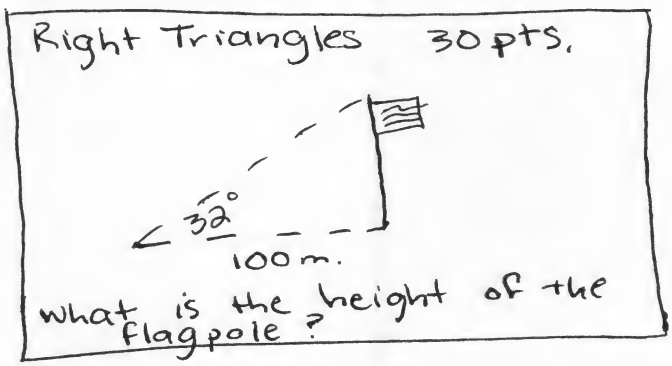
Radians	Unit Circle	Right Triangles	etc.
10	10	10	
20	20	20	
30	30	30	
40	40	40	
50	50	50	

It's Kathy's turn to choose a question.

SOME EXAMPLES:



Notice that this can be easily randomized by changing the value of θ and/or the value of r, so the game can be played over and over without repetition of questions.



This problem requires the value of tan 32°, which could be made accessible in a variety of ways.

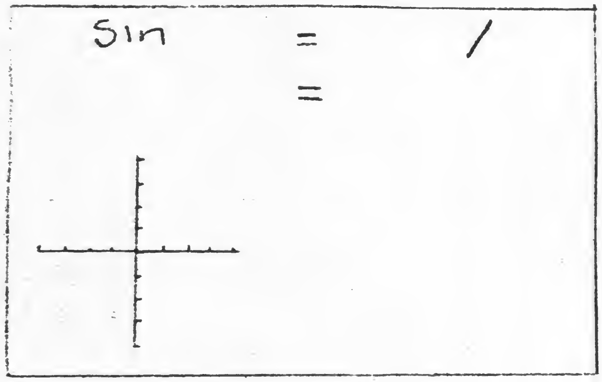
Objective: to allow students to discover trigonometric properties through guided exploration

This package makes use of discovery learning, which is useful for motivating and interesting students. Each activity would be constructed in such a way that the student would be able to experiment freely. Written material would be provided to guide the student and structure the learning process.

For examples, see the units "Sines of Special Angles," "Graph of the Sine," and "Sines Between 0° and 360°." Worksheets for this last unit are attached and give an idea of the type of support material which would be provided.

SINES BETWEEN 0° and 360° (Activity) WORKSHEET 1

1. Stop at any angle you'd like, draw what you see on the screen, and answer the questions:



a) What is the measure of your angle?

b) What is its sine?

c) What is the y-coordinate here?

d) What is the value of r?

2. We chose r=4 for this activity, but we could have chosen any r>0. Explain why the particular value of r doesn't matter.

3. Fill in the following:

sin 0° = ____________

sin 15°= ____________

sin 30°= ____________

sin 45°= ____________

sin 60°= ____________

sin 75°= ____________

sin 90°= ____________

4. Notice that the sine function does <u>not</u> increase at a constant rate. For example, there is a much larger range of angles with sines between 0.9 and 1 than between 0 and 0.1.
Look at angles between 0° and 90°:

a) For what values of θ is $\sin\theta$ between 0 and 0.1?

b) For what values of θ is $\sin\theta$ between 0.9 and 1?

1. Look at the screen for $\Theta = 30°$. Sin 30° is exactly 0.5.
   a) What is the y-coordinate at 30°?

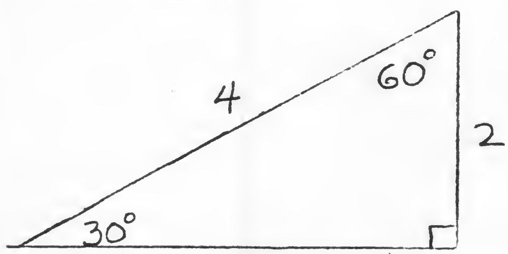
   b) What is the value of r?

   c) Fill in: $\sin 30° = \frac{y}{r} = \frac{?}{?} =$ ________

2. Suppose the value of r were changed to 10.
   a) What would the y-coordinate at 30° be, in this case?

   b) Fill in: $\sin 30° = \frac{y}{r} = \frac{?}{10} =$ ________

3. In any 30°-60°-90° triangle, the side across from the 30° angle is equal to half the hypotenuse.



Find the other side of this triangle by using the Pythagorean theorem. (Express your answer in square-root form, simplified.)

4. The relationships in a 30°-60°-90° triangle can be used to calculate exact values of sin 30° and sin 60°.

   For example, using the triangle in #3,

   $$\sin 30° = \frac{\text{opp.}}{\text{hyp.}} = \frac{2}{4} = 0.5$$

   Calculate sin 60°, in simplified radical form:

   $\sin 60° = \frac{\text{opp.}}{\text{hyp.}} = \frac{?}{4} =$ ________

   Now, use the activity on the computer to find sin 60° in decimal form: ________

1. As you go through the program, fill in the following chart. The first one is done as an example.

Θ	y	r	sin Θ
0°	0	4	0
90°			
180°			
270°			
360°			

2. From 0° to 90°, sin Θ increases from 0 to 1.
Describe the behavior of the sine function over the following intervals:
a) from 90° to 180°

b) from 180° to 270°

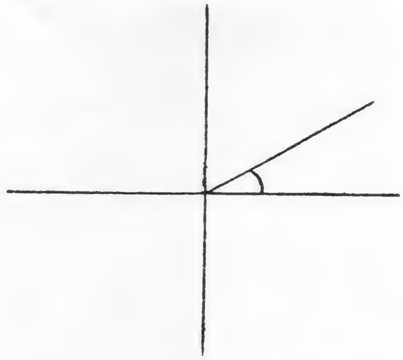
c) from 270° to 360°

3. Predict the behavior of the sine function after 360°.

4. Imagine the same activity, beginning at 0° and rotating clockwise, so that Θ would go from 0° to −360°. Describe the behavior of the sine function over this interval:

SINES BETWEEN 0° and 360° (Activity) WORKSHEET 4

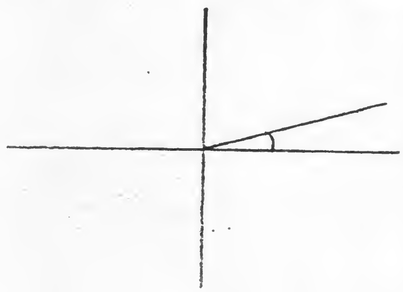
1.



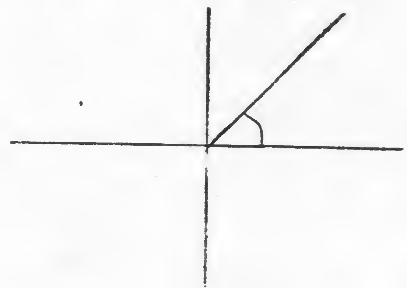
Sin 30° = 0.5.
Find another angle which has a sine equal to 0.5: ________
Sketch this angle on the set of axes.

2. Each angle between 0° and 90° has a related angle between 90° and 180° which has the same sine.
Find the related angle for each of the following, and sketch your angle on the coordinate axes.

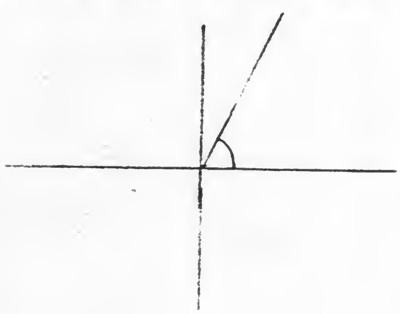
a) sin 15° = .25.. = sin ______



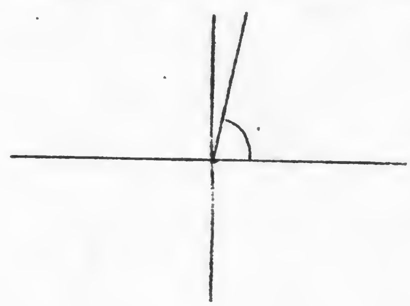
b) sin 45° = .70.. = sin ______



c) sin 60° = .86.. = sin ______

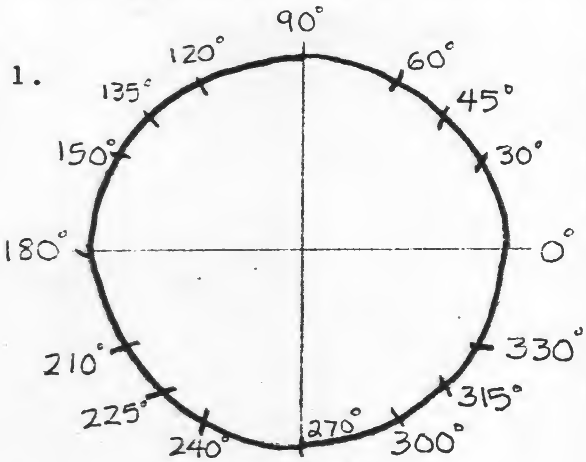


d) sin 75° = .96.. = sin ______



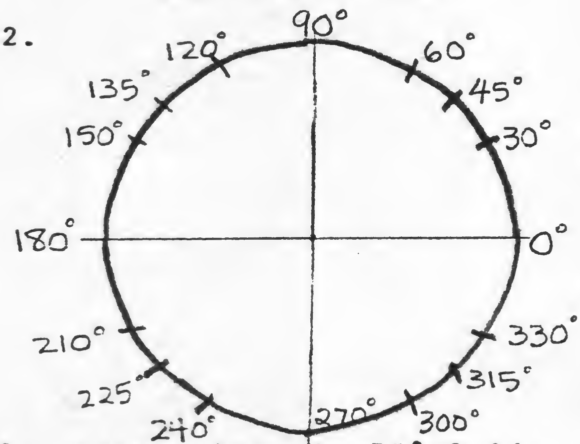
3. Describe the relationship between any two angles (between 0° and 180°) with the same sine, and explain why this works.

1.



Circle the angle measure if the angle has a sine equal to 0.5 or −0.5.

2.

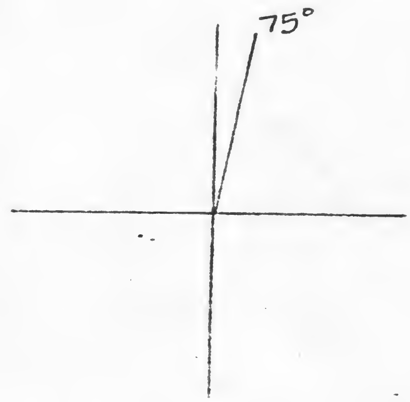


Sin 60° =0.86..
Circle the angle measure if the angle has a sine equal to 0.86.. or −0.86..

3. Given that sin 75°=0.96.., figure out the other angles which have this sine or its opposite:

__________, __________, and __________.

Sketch them on the set of axes.



4. In general, given an angle Θ between 0° and 90°, how would you find the other angles with the same or opposite sine?

APPENDIX A

PROPOSAL TO THE NATIONAL SCIENCE FOUNDATION — NATIONAL INSTITUTE OF EDUCATION PROGRAM ON MATHEMATICS EDUCATION AND INFORMATION TECHNOLOGY

Cover Page

FOR CONSIDERATION BY NSF ORGANIZATIONAL UNIT (please specify)	IS THIS PROPOSAL BEING SUBMITTED ELSEWHERE IN NSF OR TO ANOTHER FEDERAL AGENCY?
NSF—NIE Program for Mathematics Education Using Information Technology	Yes ______ No X IF YES, EXPLAIN ON ATTACHED SHEET.

PROGRAM ANNOUNCEMENT/SOLICITATION NO.:	CLOSING DATE (circle one)
SE 80-59	February 14, 1980 or August 19, 1980

NAME OF SUBMITTING ORGANIZATION TO WHICH AWARD SHOULD BE MADE (INCLUDE BRANCH/CAMPUS/OTHER COMPONENTS)

Swarthmore College

ADDRESS OF ORGANIZATION (INCLUDE ZIP CODE)

Swarthmore, PA 19081

TITLE OF PROPOSED PROJECT (Maximum 10 words)

A Computer Graphics Learning Environment in Trigonometry

REQUESTED FROM NSF:	PERIOD OF PROJECT OPERATION:
Local Contribution: $ 50,525 Total Project Cost: $ 149,802	Starting Date: March 15, 1981 Duration in months: 18

PRINCIPAL INVESTIGATOR & SOCIAL SECURITY NUMBER* (Dr., Prof., Mr., Ms.) (Name only one person)	PI PHONE NO. Area Code:
Eugene A. Klotz 559-42-6141	Office: 447-7243 Home: KI 4-2633

PI DEPARTMENT	DISCIPLINE	PI ORGANIZATION
Mathematics	Mathematics	Swarthmore College

TARGET POPULATION (Check all that apply): ☒ Older than 18 years ☐ Younger than six years ☐ 6-9 years ☒ 10-14 years ☒ 15-18 years

TYPE OF INSTITUTION: ☐ Public School System ☐ Private School System ☐ Other ☐ Public University ☒ Private University ☐ Non-Profit Corp. ☐ For-Profit Corp.

*Submission of social security numbers is voluntary and will not affect the organization's eligibility for an award. However, they are an integral part of the NSF information system and assist in processing the proposal. SSN solicited under NSF Act of 1950, as amended.

PRINCIPAL INVESTIGATOR	AUTHORIZED ORGANIZATIONAL REP.	OTHER ENDORSEMENT (optional)
NAME (Dr., Prof., Mr., Ms.) Eugene A. Klotz	NAME (Dr., Prof., Mr., Ms.) Harrison M. Wright	NAME (Dr., Prof., Mr., Ms.)
SIGNATURE	SIGNATURE H. Wright	SIGNATURE
TITLE Professor of Mathematics	TITLE Provost	TITLE
DATE August 15, 1980	DATE August 15, 1980	DATE

NOTICE OF DEVELOPMENT PROJECT
SCIENCE INFORMATION EXCHANGE
SMITHSONIAN INSTITUTION
National Science Foundation — National Institute of Education
PROJECT SUMMARY

PROJECT NO. (Do not use this space)
NSF AWARD NO.

1. NAME OF INSTITUTION (INCLUDE BRANCH/CAMPUS & SCHOOL OR DIVISION)

Swarthmore College

2. MAILING ADDRESS

Swarthmore, PA 19081

3. PRINCIPAL INVESTIGATOR AND FIELD OF SCIENCE/SPECIALTY

Mathematics (Math Education, Computer Science)

4. MAJOR DISCIPLINE CODE (See Below)*

MA

5. SPECIFIC SUBJECT(s) OR TOPIC(s):

Trigonometry

6. TITLE OF PROJECT (10 words or fewer)

A Computer Graphics Learning Environment in Trigonometry

7. SUMMARY OF PROPOSED WORD (Limit to 22 pica or 18 elite typewritten lines)

Purpose: To construct prototypes of microcomputer color graphics units in trigonometry, which would form the basis of a "learning environment." Students could explore it on their own, or work on teacher directed assignments.

Main Features: 1. Visual orientation, exploiting the full capabilities of consumer priced color graphics computer hardware; 2. user control, so that the student may browse as in a book rather than be constrained by the programmed learning format of traditional CAI; the student will also control many trigonometrical operations on the computer, and this physical involvement should reinforce learning; 3. user oriented: units will be designed for simplicity and ease of input, for readability, and for attractiveness to student users.

Intended Audience: This is primarily aimed at the standard high school trigonometry population, but special features will make it valuable for use by minorities, women, the handicapped, and older students.

Development Plan: The units will be designed and written by a team of three high school mathematics teachers and coded by a team of student programmers; the principal investigator, a college mathematics professor, will work with both teams. There will be experts in testing, psychology, CAI, art and computer graphics.

FOR NSF USE ONLY

DIVISION (OFFICE) AND DIRECTORATE	PROGRAM (NSF-NIE Program on Mathematics Education Using Information Technology)	
SCIENCE EDUCATION DEVELOPMENT & RESEARCH		
SECTION	PROPOSAL NO.	F.Y.

FOR DGC USE ONLY

START AND END DATES	AMOUNT GRANTED

*Major Discipline Classification: Please classify the proposal according to the following major group(s). The appropriate code should appear on line 4 of the Project Summary Form.

BZ	(Biology): subjects in category of "Life Sciences."	MA	(Mathematics): other than Computer Science
CH	(Chemistry)	PY	(Physics): includes Physical Sciences
EZ	(Earth Sciences)	SZ	(Social Sciences): subjects in category of "Social Science"
IN	(Inter- and Multidisciplinary Sciences)	ZZ	(Other Sciences)

1. DESCRIPTION AND SIGNIFICANCE OF APPROACH TO BE DEVELOPED

Introduction: Trigonometry represents an entry into higher level thinking for many high school students, both in terms of increased deductive demands and as the first time they have had to deal with algebra and geometry together. It is discouraging for some students who have previously been good in math [1]. Moreover, although trigonometry used to be a full semester course, it is now often relegated to half a semester or less. A recent study at Purdue University indicates that trigonometry is either being slighted by many high schools or taught in such a way that university students required remediation [2]. Last year, engineers at the University of Illinois complained about the substantial and increasing need for remedial work in algebra and trigonometry, especially the latter [3].

Some scholars believe that microcomputers, because of their low cost, power, and versatility, will lead to a new era of computer application in education [4]. However, computer aided instruction* has not been without its problems in quality. For example, in a recent review of over 4000 CAI programs, only about 3-4% were found acceptable by faculty in the fields concerned [5]. A current survey article concludes that "the single most critical issue in CAI today is the development and sharing of quality CAI materials" [6].

It is the goal of this project to develop prototypes of high quality interactive computer color graphics units in trigonometry. The appropriateness of using this technology to deal with this area should be apparent: computer graphics, color, and motion are powerful tools with great possibilities for pedagogy through visual interest. According to a recent article in Creative Computing "for those of us who could never get past trigonometry as taught in the schools... the personal computer and its graphic display offer new hope for understanding" [7].

Approach: We propose to develop a number of interactive computer graphics units in trigonometry. These would be quite small, "standalone" modules, not lengthy tutorial lessons: "adjunct CAI" as opposed to complete "courseware". We would hope to develop

*We shall use CAI to stand for the full gamut of instructional use of the computer. Some authors use it more restrictively to denote only drill-and-practice programs.

examples of a broad variety of types of topics: definitions, formal derivations, and examples and applications from the real world, science, and mathematics. All would feature considerable student control and interaction. The focus would be on review and enhancement for grades 10-12, but they could be used for remedial purposes, enrichment, and possibly even for first contact instruction.

Each unit would have an associated quiz module which the student could request before trying the unit and would be automatically administered after the unit was completed. There would also be a diagnostic unit which would sample from the quiz modules, which the student could request at any time. Usually quiz questions would not be multiple choice. Questions would often involve visual activities regulated by a joystick,* for example in measuring angles or in plotting points in polar coordinates. (Such frequent testing can apparently lead to improved student performance and does not increase anxiety, at least if designed for success [8].)

When possible and when time permits, multiple approaches would be taken to the same topic. For example, one unit might treat sines and cosines as coordinates of points on the unit circle, while another dealt with sines and cosines as ratios of sides of right triangles.

Our basic orientation could be described as visual, user controlled, and user oriented. By visual orientation we mean the use of color graphics as an important pedagogical tool. The use of color will clarify complicated figures, allow added emphasis in figures and text, and contribute significantly to visual interest. We plan to use the graphics in a manner which is esthetically pleasing and entertaining, but not distracting.

The entertainment aspect is particularly important for the usual trigonometry age group, since a number of studies have shown that when students reach secondary school their positive attitudes towards mathematics begin to decline [9]. Moreover, we postulate that an entertaining visual approach would help combat anxiety, which all

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*a lever similar to the pitch/yaw control joystick in a small airplane. A joystick can be used, for example, to point to a particular spot on the screen; much more sophisticated uses are to be found in many video games.

studies indicate is negatively correlated with achievement. Moreover, there exists a small but significant correlation between mathematics achievement and spatial visualization [10] and we would expect the latter to be improved by our visual approach.

In particular, our visual orientation is meant to contrast with the verbal which permeates so much of education. Such text as we use will be engineered for readability: there will be lots of blank space, short lines (by phrases, not right-justified), and the student will be able to control the rate of presentation. Whenever appropriate, the student will be given graphical tools (e.g. the ability to construct a unit circle by specifying its center, or the ability to lay down a number line by specifying the position of 0 and 1), and will be able to control these tools with the joystick.

The graphical tools also tie in with our notion of user control. This would further encompass control of level of discussion when enrichment or further clarification are appropriate. For example when radian measurement of angles is introduced, an asterisk on "counterclockwise" would allow the curious to discover that tradition is the only reason why this direction is considered positive, but lack of adherence to this convention can result in negative signs appearing in various physical formulas.

We would even place various levels of rigor under user control; as the presence or absence of this sort of formality does not seem to have a strong effect on student achievement [11], it would seem to be more effective to leave such matters up to the student's own level of curiosity. For example, a unit on the cosine of the sum of two angles might present the formula and point out that a derivation is available. If the student chooses to follow up on this the appropriate circle with two chords would be displayed, the endpoints labeled, and it would be pointed out to the student why it suffices to establish the equality of these chords. Those still curious would be presented with an argument for the equality, while the others could return to the portion of the unit dealing with mastering the formula.

As a final example of student control, there would be no rigidly controlled routing of students (although suggestions would be made by the control program, and each unit would have available a list of concepts (apparently) needed for its mastery). We wish to provide a learning environment in which the student can browse at will. Such student

control of learning route seems strongly preferred by students [12]. Of course, a teacher would often make assignments or suggestions of particular units to be covered.

Finally, by "user oriented" we mean that we would take pains that the student was not overwhelmed by text, lists of options, or input demands. The latter would typically be by joystick, simple numbers, or single letters (with easy access to a diagram showing the location of possible letter options). We would also insist upon ease of entry, exit, and branching, upon readily available help programs, and browsing capabilities. We are aware that many persons are ill at ease and reluctant to use computers at first; we will take pains to design a New User's Unit which will allay fears and provide a pleasant and intriguing introduction.

From a psychological point of view our approach is intended to build a base of mathematical intuitions through multiple representations of the same mathematical concepts. By combining vivid graphical displays, motor control on the part of the student, and algebraic symbols and formulas, we hope to create a deeper sense of mathematical relationships than many students are able to derive from traditional presentations. Our reliance on multiple modes of representations is deliberately reminiscent of the psychological and representational themes of J. S. Bruner who has suggested that an individual's understanding of physical, logical and mathematical relationships is built on three kinds of experience and internal representation: enactive (motor), iconic (perceptual), and symbolic (linguistic or algebraic) [13].

A number of our ideas would seem to unite good pedagogy with simplicity of programming. However, if for example, others find it desirable to embed the units in a hierarchy controlled by the teacher (or by a program constructed in accordance with some generative CAI principles), the modularity of our programming will make such restructuring relatively simple.

<u>Examples to be Developed:</u> Selection will be made within areas of known student difficulties, and as befitting a pilot project, to illustrate a variety of pedagogical approaches, and from a broad range of trigonometrical subjects. Examples of possible candidates include: measurement of angles, definition and properties of the usual trigonometrical functions, why the length of the circumference of a unit circle is 2π, the

sum formulas, trigonometrical identities, trigonometry of triangles, trigonometry and navigation, inverse trigonometrical functions, polar coordinates, etc. For example, since a number of students find polar coordinates the most difficult part of a high school pre-calculus course, we are very likely to develop units on this subject. For a flavor of what we have in mind, see the Appendix, page

<u>Final Product:</u> The final output of the present project will be a number of interactive color graphics units on trigonometry driven by a control program. These will have been debugged, well documented, and tested by a variety of user groups. They will be ready to run on the hardware upon which they were developed, and considerable thought will have been given to dissemination and transportability. Although a thorough treatment of all of trigonometry will not be possible in the given time period, it is hoped that enough modules will be developed to make the entire unit immediately of some pedagogical value, and to form the basis for a more complete learning environment.

<u>Special Concerns:</u> Although our specific target population is high school students who are taking a course containing some trigonometry, our approach should also be of particular value to a number of special populations. We would expect that our emphasis on the visual rather than the verbal would make our units of special value to those who are weak at standard English. Our insistence on very simple input would be of use to some handicapped persons, as would the moveability of the standalone hardware upon which the programs will run. Since the units would not be embedded in large and elaborate courseware, they would be well suited for review for older students returning to high school or college (especially since the subject is an important prerequisite for calculus, which is one of college's more pervasive and difficult requirements).

There are at least two directions in which our project may be of special value to women. One has to do with our approach which stresses perceptual rather than formal-logical aspects of the subject. In at least one study [14] such an approach allowed girls to reach a level of performance in a perceptual skill in which they are generally assumed to perform less well. Secondly, the findings seem to indicate that girls show both more math anxiety in general, and test anxiety in particular than boys [15]. We would expect that our approach would be anxiety relieving, for such reasons as its high

esthetic and entertainment component. Also, various features of the program should combine to give students a sense of power and control over the subject and also alleviate fear of computing machinery. Moreover, we would expect that frequent and simple testing would help relieve test anxiety.

Past and Present Activities in Applications of Computers to Trigonometry: A search was conducted to determine the extent of current or previous activities along the lines envisioned in this project. The following books were consulted: NSF Source Book of Projects, Science Education Development and Research, F.Y. 1979 (with references to earlier years); Hoye, R. E. and Wang, A. Index to Computer Based Learning. Abstracts of all ERIC documents relating to trigonometry were examined. Telephone conversations were held with highly placed individuals in the following organizations: California State University at Fresno "ABC's of CAI" Project, Minnesota Educational Computing Consortium, New Jersey Educational Computing Network, CONDUIT, EDUNET, MicroSIFT, and the Educational Technology Center (University of California, Irvine). The search revealed no past or present activities concerned specifically with trigonometry via computer graphics. Although there were no current trigonometry units of any sort known to the persons at the above institutions (which are concerned with keeping abreast of what is available), the Index to Computer Based Learning did have 15 entries devoted to trigonometry. Most of the units were quite small, quite old, and many were written in Coursewriter (an archaic and particularly procrustean author language); none were at all close to what we are planning. Both SOLO and very large projects PLATO and TICCIT apparently have some trigonometry components, as does the CONDUIT drill-and-practice algebra unit (with no graphics). All relevant CAI which we can examine will be considered with great interest to see what we can learn for our very different approach, even though "estimates of great numbers of buried treasures of classical CAI lessons [are] mere fantasy" [16].

Throughout its brief history, although there are some instances of high quality work, CAI has not fulfilled its promise. The previously quoted statistic of 3-4% acceptable out of 4,000 programs speaks for itself. Until very recently, hardware was expensive and/or unsatisfactory. Software in CAI is rather complicated but has often been constructed without the helpful and simplifying computer science tools developed

in the 70's, and is frequently poorly documented.

Moreover, CAI development has tended to follow two paths. The first consists of an author-programmer who is able to take full advantage of the power of the computer by using a standard high-level language, but who tends to lose sight of the input capabilities of the student and other aspects of good pedagogy. The student is overwhelmed with text, options, and heavy input demands. At the other extreme is the author who makes use of one of the simplified author languages, and is thereby cut off from utilizing many of the computer's capabilities. "Such simplistic CAI produces limited results" [17].

Finally, in "classical" CAI an attempt is made to supplant rather than supplement many of the activities of the teacher. This calls for considerable expertise available at all times on the psychology of learning, and for much sophisticated testing. Since one wishes to construct a master program to route the student according to past and present responses, and since student responses come in amazing variety, this calls for large and complicated programming and long and difficult debugging. If one wishes the program to be able to carry on a reasonably natural dialog with the student, this adds further burdens. In short, this type of CAI is a formidable undertaking. No wonder that many projects and persons have fallen short.

Likelihood of Success: There are three principal reasons we see as to why we should be able to construct high quality CAI material in trigonometry: approach, personnel, and milieu. Our basic approach will be in two working units: the teachers (who will approach production from the viewer's perspective), and the programmers (who will approach the teacher's plans from a top-down, structured point of view). The principal investigator will serve as go-between (sometimes wearing either hat), and the consultants will fulfill roles outlined below.

The teachers will initially be armed only with a basic knowledge of the graphic capabilities and restrictions of the hardware, their pallette. They will be asked to design exactly what they wish the student to see, and to be daring, visual, nonlinear, and non-textbook in their approach. The total approach will encourage the development of units which are viable mathematically, pedagogically, and visually. The approach of the programmers, with its concomitant stress on documentation, should result in more rapidly

created, flexible, easily amended, and transportable software, with fewer errors.

See Section 3 for a discussion of the strengths and mix of personnel. For the present we note that our mix is similar to that of a very successful, relatively small scale CAI group, that of Professor Alfred Bork, University of California, Irvine. While his group is larger, his goals (in particular regarding dialog) are much more ambitious. By focusing on simple input and user control, we have considerably simplified our problems and will be able to concentrate on the medium and the mathematics.

There are considerable strengths in our milieu. Swarthmore College and the Swarthmore High School are small and have considerable interaction (for examples: the chairman of the college mathematics department is on the school board; the principal investigator and one of the high school teachers served on a recent evaluation of the district's mathematics curriculum). The college and high school have pioneered in the experimental teaching of calculus via the computer (and in a computer language, APL). A number of college math department faculty have taught in enrichment programs at the high school. Both the college and the high school are seriously committed to using the computer in education. The College's Program in Education, directed by Professor Travers, forms a strong and continuing relationship, and one which has involved the only non-Swarthmore teacher. The new director of the College's Computing Center, Elizabeth Little, is very supportive of this project, and offers considerable expertise and perspective. All proposed staff and consultants (except Alfred Bork) are located in or near Swarthmore and most have previously worked with a number of the others.

Intended Audience: Our primary audience consists of trigonometry students in need of review, remedial work, enchancement, or makeup work. We also hope the learning environment will be of value to other audiences. We will test units on minority students,. 7-9th graders (for enrichment), and persons taking calculus (for review and remedial work). We will not be able to make special tests on women, older students, or the handicapped, although we believe our approach may be of special advantage to these populations as well.

How the Audience Will Be Affected: It is our intention that the learning environment further the learning and review of trigonometry, making use of techniques and learning tools not currently available. In particular, we expect the audience to be especially

affected visually and through the use of motor control. We hope to produce something which will delight users, thus affecting postively their attitude toward mathematics.

How the Project Can Improve Mathematics Teaching and Learning Processes: It is hoped that the completed learning environment will give teachers a new and powerful tool, and that the units finished by this project will convincingly display the capabilities of the new technology.

It is to be expected that many small groups will wish to attempt CAI because of the advent of low-priced microcomputers with color graphics capabilities. However, the production of good quality CAI has been difficult even by large groups. We believe we have some understanding as to why this has been the case, and that we have developed a well-articulated and relatively simply approach which is suited to "cottage industry" CAI. We hope that our successes and failures (tested and written up for newsletters, special interest group publications, and for reports to NIE and NSF) will be of help to others who wish to produce good CAI with modest resources.

The completed trigonometry learning environment should improve the learning of mathematics in contexts other than schools, since the necessary hardware will be a modestly-priced standalone microcomputer system. Such contexts might include libraries, homes, and hospitals.

2. PROCEDURES

Procedure for Assessing Our Basic Orientation and Its Consequences: While we believe that our basic approach is sound, if somewhat novel, we do not believe it incapable of improvement. Before beginning construction of the units we will undertake a serious assessment of our basic orientation and its consequences. Our consultant psychologist will review for us the relevant literature in cognitive psychology and learning theory. The teachers will have increased their perspective and background in CAI by trying material developed elsewhere, for example the PLATO materials at the University of Delaware. Further, we will avail ourselves of the information on research and development available through CONDUIT and, it is hoped, from MicroSIFT. We shall also have outlined our plans in the newsletter of the Association for the Development of Computer-Based Instructional Systems, and would hope to receive helpful comments from that source. We will continue to attend

appropriate meetings, such as those sponsored by NECC, ADCIS, AIDS, and the various ACM SIGs, in hopes of fruitful interchange with others working in the area. All in all, we would expect to carefully scrutinize our basic approach on several different occasions, including just prior to beginning the actual construction, with Professor Bork after we have produced viable units, and after receiving the report on the classroom testing.

Choice of Trigonometry Units: We will go through a variety of texts searching for topics and approaches. For further ideas we will search through a number of relevant journals, including The Mathematics Teacher, The Two Year College Math Journal, Mathematics Magazine, and The American Mathematical Monthly. Trigonometrical topics will be ranked by the high school teachers according to student difficulty. This ranking will be consulted in all further choices, with topics perceived difficult given precedence over the others.

The initial selection will be of 2 or 3 topics which are simple and relatively straightforward, and which could be programmed with the graphics tools thus far developed. (In general, we expect our graphics techniques to lag behind the teacher's desires; the academic year will serve as a catchup period). After experience has been gained with these units, others will be selected of varying complexity, to develop a broad spectrum of prototypes.

"Authoring" Procedure: Initially the teachers will be given a notion of the absolute limitations of the hardware, but stress will be placed on appreciating its capabilities. To appreciate hardware capabilities, the teachers (and programmers) might spend a few hours at an arcade playing video games on what is essentially the same hardware as that chosen for the project (N.B. although we eschew monsters from outer space as any help in learning trigonometry, the underlying graphics techniques are highly developed).

The teachers, usually working in teams, will plan the units. They will make very rough sketches of the basic graphic output and will indicate the branching logic on large sheets of paper. They will make more careful sketches on paper approximately the size of a 19" TV screen, using crayon to approximate the degree of resolution. When the sketches are ready for coding, the teachers will talk them through with the programming staff, whose suggestions or objections might occasion some revision. At least at the outset, the teachers (except for the principal investigator) will not be further involved

with the programming except when insurmountable difficulties or disaster occur. After a number of units are developed, when the teachers feel confident in not tainting their author role, they might assist in some of the programming.

<u>Selection of Student Programmers:</u> There will be a pool of Swarthmore students who would be suitable for our project; for example, the principal investigator is teaching an introductory computer science course which emphasizes structured programming and good documentation. However, at least for the first summer to assure a full complement of programmers of the highest quality (and perhaps even with skills in working with graphics on our hardware), we shall advertise at selected eastern seaboard colleges. This procedure has been followed here with excellent results in getting programmers for undergraduate research projects.

<u>Programming Procedures:</u> By the time the programmers are ready to begin, the teachers will have amassed a list of basic graphics needs. Because of the nature of the subject it will not be necessary to consider such formidable difficulties as the three dimensional hidden line problem (or probably very much that is three dimensional at all). The programmers will begin work constructing graphics modules, input/output programs, and any other background subroutines which appear necessary. One of our consultants has considerable experience with the graphics hardware and software of the microcomputers we shall be using; he will get the programmers started on coding the graphics. When programming difficulties arise, they will also have available the assistance of the college's Computing Center Staff.

All coding will be done from a top-down, structured approach, with good documentation. The evaluation and documentation criteria of major non-commercial assessors and distributors of CAI software will be kept before us and aimed for (these include: MicroSIFT, CONDUIT, and the California State University at Fresno "ABC's of CAI" project). When graphics is involved it is impossible, at least at this time, to construct software which is immediately transportable to different hardware, especially if one wishes to use powerful features found on a specific machine. We shall work with this problem in mind, and shall endeavor to minimize the difficulty.

Testing Procedures: The first units will be informally tested on a number of faculty and students; their comments will be used to assess and rework these units as necessary. Professor Eva Travers will develop questionnaires to be administered to students who next use the units. The studies will be attitudinal; the scope of our program does not allow for statistical studies to measure the effectiveness of our material. (One might note that there are those in the computer-based education field who argue that attitudinal studies can be more meaningful than statistical, anyway [18]. The questionnaires will be read through by appropriate personnel for immediate feedback. The questionnaires will also serve as a basis for more formal study by Professor Travers, and this study will be discussed and considered by the teachers.

The questionnaires will be administered by a college student aide, under the training and supervision of Professor Travers. They will be given to a selection of (1) high school students who try the units as part of their course work in Precalculus Mathematics, Algebra II, or in calculus; (2) college students who try the units as part of their precalculus course, or as review in the calculus courses; (3) Upward Bound students, who are part of the program for minority students centered at Swarthmore College; (4) students (primarily minority) who are part of the College's pre-enrollment program, (5) students in grades 7-9 (as part of an enrichment study at Germantown Friends School).

We wish to find out several kinds of information from the questionnaires: areas in need of improvement in specific units, general techniques which are going well or poorly, variations among the populations in response to techniques or particular units. As mentioned above, we have no hope of definitive answers to any of these questions, but we should be able to amass some useful and suggestive results.

As the student aide assists a student in trying a unit, he will note where and how the student asks him for help. We will use this information as the basis for constructing "help" subroutines which students will be able to call when they are in need of assistance.

As soon as we have an appropriate amount of material developed, we will avail ourselves of the offers for informal peer review extended by persons at CONDUIT and at the New Jersey Educational Computer Network. We will be in contact with MicroSIFT and will also submit materials to them as soon as appropriate.

The units will thus be tried and tested by a number of different kinds of users (from 5 student populations as well as faculty and CAI professionals). Reworking and restructuring will be done in response to trial results and criticism. This should do much to assure the production of a useable product, especially since the teachers all hope to use the material in their classes. To assure that the units are in fact used, we shall build in an "attract" mode which runs through some of the more visually stimulating portions, and to which the program automatically reverts whenever it is left unattended. We have already mentioned our commitment to constructing a really effective New User's Unit whose purpose is to make students comfortable in using the program.

Swarthmore College, the proposed grantee, will lend several terminals to Swarthmore High School, so that the units can continue to be part of the curriculum there.

Procedure for Obtaining Permission from Schools: At the schools we have begun discussions with administrators toward obtaining approval for pre-college student's participation in project activities; at the College we have discussed obtaining the necessary certification regarding research involving human subjects, as reviewed by the Research Ethics Committee. No difficulties are foreseen, especially since the activities involved are not at all controversial, and all institutions involved allow appropriate research activities of comparable scope. Moreover, student participation will not be required in any activity, no confidential data will be used or generated, and the high school activities will involve classes taught by teachers on the project. The responsibility for obtaining official approval from the various institutions has been apportioned to appropriate project members, and we expect formal approval early in this academic year.

Procedure for Making Software Available: As mentioned above, we plan to keep close ties with both MicroSIFT and CONDUIT, and would aim to produce a complete learning environment which they could distribute. Indeed, we would hope to continue the project (with or without NSF-NIE support) until this was possible.

3. PERSONNEL QUALIFICATIONS

The Principal Investigator, Professor Eugene Klotz, received his Ph.D. in Mathematics from Yale University in 1965. He has worked with computers since 1956, and has used computers and computer graphics as course enrichment since the late 1960's.

Although not formally trained in computing, the principal investigator has attended a number of workshops and conferences devoted to computers and computers in education. In particular, the Eastern Pennsylvania and Delaware Section of the Mathematical Association of America (for which the principal investigator has served as both Chairman and Vice-Chairman) has presented four special sessions devoted to computers or computer graphics in the last five years; the principal investigator was instrumental in organizing several of these sessions. He recently attended an IEEE workshop in CAI. Much of the principal investigator's background in computers has come from actually using them, and from the preparation for and teaching of various computer-related courses at Swarthmore: Introduction to Computer Science, Data Structures, Discrete Mathematics, Applied Modern Algebra.

The principal investigator has a long-standing interest in students who have difficulties in mathematics. In 1978, he organized and taught Swarthmore's first course for students unprepared for calculus. (Such courses are standard at most colleges, but Swarthmore had previously ignored this class of student). As preparation for the course, the principal investigator did an extensive literature search on mathematical difficulties of students, "math anxiety", and the special mathematical problems of women and minority students. One of the fundamental subjects in this course is trigonometry, and the principal investigator now has several years experience in teaching this essentially high school subject.

The principal investigator has had other involvement with high school mathematics. He has taught in-service courses for high school mathematics teachers, he was a member of the recent committee charged with evaluating the District's mathematics curriculum, and two years ago taught the high school calculus course for a week.

A National Science Foundation CAUSE grant to Swarthmore College gave the principal

investigator the opportunity to develop some CAI material from 1976-1978. Completed programs had to do with symbolic logic, Venn diagrams, polynomial multiplication and finding the roots of polynomials. Each program presented the student with graded, randomly generated (or, in two portions, randomly selected) mathematical problems, with hints, help, and answers readily available. The units were primarily drill-and-practice, but there were a number of tutorial features as well.

Although these first CAI units were useful, certain hardware and software problems dampened their effectiveness. Moreover, the operating system was changed for the time-sharing system on which the programs were running (the PI is now very sensitive to problems of transportability), and the extreme complexity of the programs has made the program changeover very cumbersome (the PI is now very conscious of the value of structured programming). The PI has spent some time studying how to make his CAI units more effective, and he has had the opportunity to try examples of the 4% (at a liberal estimate) of CAI which is of high quality, and to examine the philosophy behind this CAI. The results of his study are embodied in the basic approach articulated in the first section of this narrative: visual orientation, user control, and user oriented. To this should be added that to be really effective, it appears that one should not have too small an operation: his previous CAI was developed with at most two student programmers, and this did not allow for the necessary perspective and distance between author and programmer, nor did it make for real efficiency of operation.

The PI has both a strong interest and considerable experience in good management and organization. For example, last year he chaired a joint Mathematics-Engineering Departments Committee charged with developing a curriculum in Computer Science (and later asked to collect additional information regarding College computer use). In 6 months this committee was able to produce a written report on its primary responsibility, issue position papers on computer languages, and on mini programming courses, and survey: facilities and curricula at comparable institutions, Swarthmore alumni working in computing, recent alumni in Computer Science, requirements of leading graduate schools in Computer Science, Swarthmore faculty who use the computer (this latter by personal interview). The surveys were processed, summaries were issued, and reports made which drew conclusions from the

summaries. Moreover, the PI remains on very cordial terms with all members of his committee, and new courses in the Computer Science curriculum will be available to students this fall.

The PI has a longstanding interest in teaching with a visual approach, and has served on the Committee on Educational Media of the Mathematical Association of America. For many years he has given considerable thought to visual presentations in his courses, often handing out drawings (many computer generated), and routinely using colored chalk for emphasis and for further drawings.

Joseph Blass has a Masters in Mathematics Education from Temple University. He has taken considerable course work beyond the Masters, including 9 credits in Computer Science. Mr. Blass has taught at Swarthmore High School for 9 years, and is currently Chairperson of the Mathematics Department. He regularly teaches Algebra II and Precalculus Mathematics, both of which cover various aspects of trigonometry. This year he is offering a new course, Introduction to Computer Programming for 8th graders. Mr. Blass has a special interest in computers and a main goal of his is to make further computer material available to high school students.

Cynthia Schmalzried received her B.A. in Mathematics from Swarthmore College in 1976. She graduated with Distinction, and brings to the project strong mathematical skills. Ms. Schmalzried is interested in trying the units as enrichment in various courses in grades 7-9, as well as using them in the trigonometry section of the 10th grade Algebra II course which she teaches.

Ms. Schmalzried has taught at Germantown Friends School since 1976. While at Swarthmore, she student-taught in Ms. Stuppy's class, under the supervision of Professor Travers. This year, Ms. Schmalzried served on the Mathematics Education Self-Evaluation Committee at Swarthmore College.

Regina Stuppy obtained her B.S. in Mathematics from the University of Illinois in 1970. She has continued her education by taking graduate courses at Temple University and at West Chester State. Included among these courses are one in individualized instruction and one in CAI. As part of the latter course, she examined several CAI systems and wrote some CAI material.

Ms. Stuppy has been involved with the computer since high school, where she learned FORTRAN as part of a summer NSF program. She has used computers for class demonstrations for some years and has participated in in-service training sessions on the computer in the classroom.

Ms. Stuppy has been at Swarthmore High School since 1971, and was a member of the recent committee which evaluated mathematics education in the school district. She teaches precalculus and calculus courses, the first of which covers trigonometry and the second uses it.

Ms. Stuppy is particularly interested in the problems of women in mathematics. In 1978 she wrote a paper on this subject and gave it at the annual meeting of the Pennsylvania Council of Teachers of Mathematics; it was subsequently presented at the annual meeting of the NCTM.

Professor Eva Travers obtained her Ed.D. in 1973 from Harvard University, specializing in Social Studies and in Learning Environments. She has been at Swarthmore College since 1975, and has directed its Program in Education since 1977. She serves as administrator, and is the liaison with Swarthmore High School. She supervises student teachers, and in this capacity has worked with both Mr. Blass and Ms. Stuppy.

Professor Travers has considerable background in evaluation techniques: instrument development, data collection, analysis, and feedback. She has served as a resident evaluator at an alternate school for two years and has much experience with non-traditional learning environments. She is solidly grounded in education theory, and has both first-hand and theoretical background in how adolescents learn in classrooms.

All of the expertise mentioned here will be useful to the project. In addition, since she describes her knowledge of trigonometry as far from current, she will be able to try the units from the perspective of the user.

CONSULTANTS

Alfred Bork is Professor of Physics and Information and Computer Science at the University of California, Irvine. He has developed graphic CAI dialogs of the highest quality, and has written extensively on the production of the CAI software. Professor Bork has recently founded the Educational Technology Center, which is devoted to the more

effective and efficient use of information technology in education. He is a series editor for CONDUIT, an editor of The Journal of College Science Teaching, and was recently the American Association of Physics Teachers' Millikan Award Lecturer.

Theodor H. Nelson received his B.A. from Swarthmore College in 1959 and obtained an M.A. from Harvard in Sociology. He is the editor of Creative Computing Magazine, and director of Project Xanadu (which is in the final stages of developing a high-powered linked-text storage and retrieval service). He has designed several graphics systems, consulted, lectured, and written extensively on a wide range of computer topics, and is the author of two books, The Home Computer Revolution, 1977, and Computer Lib, 1974. In the latter he anticipated many of the ideas on CAI which have been worked out by the PI in the last two years, and which are articulated in Section 1. For 20 years Mr. Nelson has specialized in the design of interactive screen systems, and is well known for his point of view stressing art and conceptual unity.

Mr. Nelson has taught at a number of colleges, including Swarthmore College, 1977, [illegible]d in both Art and in Computer Science at the University of Illinois, Chicago Circle. He is one of three invited speakers from the U. S. at the International Computer Conference (IFIP) at Tokyo and Melbourne this year.

Jeffrey Travers received his Ph.D. in Psychology from Harvard in 1970. His dissertation was on perceptual processes in reading, and he used CRT displays to create the visual patterns in his research. Dr. Travers taught Cognitive and Developmental Psychology at Swarthmore for six years. Most recently, he was staff director for the Committee on Child Development Research and Public Policy for the National Academy of Science.

Dr. Travers has a longstanding interest in applying psychological principles to the learning of technical subjects, such as mathematics and science. In the Spring of 1979 together with a physicist, he taught a course on Special Relativity to Harvard freshmen with no particular mathematics background. Aside from wishing to teach advanced physics [illegible] relatively naive students, both Dr. Travers and the physicists were interested in examining the process whereby mathematics and science concepts are learned. Their special focus was on the representation of systems and, for example, they found that teaching the students affine geometry was of considerable value.

The PI finds that Dr. Travers is particularly helpful both because he has broad knowledge and because he can communicate it lucidly and palatably to non-psychologists. He will be able to supply the needed information on learning processes both from his own knowledge and through discussions with specialists, with many of whom he has contact.

PROGRAMMERS

It should be noted that there are high quality CAI operations (such as Professor Bork's) which rely primarily on students for programming. Because of our selection procedure, we will have students who are up to the task.

4. ORGANIZATION AND MANAGEMENT PLAN

The personnel for this project can be divided into four basic groups:

- the teachers (high school and the principal investigator) whose main function is to "author" the units;
- the student programmers (4 each summer and up to 4 part-time during the academic year) who do the coding;
- the student evaluation team (the faculty member in Education and a student aide) who set up, facilitate, and help in student use, who administer the questionnaire and provide immediate and formally studied student feedback;
- the 3 consultants who offer needed expertise in learning theory, computer-based learning, computer graphics, art, and computer-user interaction.

In addition to fulfilling their separate roles, all these groups interact at various times, primarily on setting general policy.

In order to clarify the working structure of the project, we present first a list of the major responsibilities of each of the participants. Then, to show how the activities tie together, we give a schedule of activities.

Responsibilities of the Principal Investigator (Hereafter: PI)

Overall: The PI is responsible for the general management and organization of the project. He makes policy decisions, and decisions regarding the basic approach, after due consultation with the entire staff. He will conduct the correspondence with other organizations

and persons, order equipment, etc. He will write the reports for the funding agencies. With the Student Programmers: The PI will make sure they understand the desired structured approach to programming, and adhere to clear documentation standards. He will see that they understand what is expected in the various programs. He will answer their technical coding questions, or make sure they have someone to turn to (consultant or Computing Center) who can. He will apportion the coding tasks, and supervise as necessary. With the Consultants: The PI will make sure they understand what is desired of them, plan timing with them, and make any necessary arrangements for them.

Responsibilities of the 4 Teachers (3 high school plus PI) Spring, 1981: Fill in background through reading and through trying out other CAI; select necessary graphics functions. Thereafter: write the trigonometry units and revisions thereof. Special Responsibility of Joseph Blass (Chairman of Swarthmore High School Mathematics): He will serve as liaison with the high school and the school district. In particular, he will attend to the requirements regarding pre-college students and experimental curriculum development projects, as set forth on p. 10 of the program announcement. Special Responsibility of Regina Stuppy: She will search for special ways in which the units can be made attractive to women, and will see that the language and other aspects of the units are not off-putting to women. Special Responsibility of Cynthia Schmalzried: She will fulfill the same functions with Germantown Friends School as will Mr. Blass at Swarthmore High School. In addition, she will oversee testing the developed material on 7-9th grade students, and will write a short report of the results for the final report.

Responsibilities of the Swarthmore High School Teachers: In the spring of 1981, they will search the literature on trigonometry, make up a list of possible topics, and rank these topics according to perceived student difficulties.

Responsibilities of Professor Eva Travers (Department of Education): Throughout the project, Professor Travers will provide background expertise in how students learn in classroom situations. She will try out the developed units herself, and will participate in the major reassessments of the basic approach. During the summer of 1981, she will develop an attitudinal questionnaire to be administered to a sample of students after they try a unit. She will also train the student aide who administers the questionnaire. In the

late spring of 1982, she will make a study of and report on the results of the questionnaire.

The Student Aide will work with Professor Travers as indicated above. The aide will also work with Professor Travers and the PI on developing his "helping" role. During the academic year and the summer of 1982, he will assist a sample of students to use the units, and then administer the questionnaire to the students.

The Student Programmers will be responsible for coding and debugging the background subroutines and the programs whose front ends are developed by the teacher group. They will follow structured programming principles, and in particular will produce sound documentation. They will work under the supervision of the PI, with basic instruction in graphics from consultant Theodor Nelson, and with assistance from the College's Computing Center. During the two summers, a chief programmer will be selected from among the students to help with supervisory activities.

Responsibilities of the Consultants: Theodor H. Nelson will work with the PI and the programmers in getting started on the graphics. Mr. Nelson will also work with the teachers, pointing out the graphics capabilities of the hardware and helping them understand its limitations. He will make suggestions to enhance the user-computer interface. In general, it is hoped that he will help keep the project from becoming so narrow and academic (in the bad sense of the word) that we fail to use fully the capabilities of the new technology. His period of greatest activity will be from late May through July, 1981, but he will be consulted occasionally thereafter. Dr. Jeffrey Travers will review the psychological literature for relevant information from the cognitive sciences, which he will summarize and present to the staff in late June, 1981. He will have been apprised of our objectives and of our basic approach and its consequences as we see them; he will discuss how the literature suggests we could improve the latter toward achieving the former. Dr. Travers will also advise us on color combinations to avoid for the color-blind. As the budget permits, he will try out the units we develop and participate in later reassessments of our basic approach. Professor Alfred Bork will visit the project for two days in late August, 1981, after we have had the opportunity to develop some units in accordance with our basic approach. He will criticize our units, comment on the approach we have developed, discuss his own philosophies, and demonstrate some of the software he has produced.

SCHEDULE OF ACTIVITIES

<u>INITIAL PREPARATION</u> March 15–June 20, 1981

Goals: Teachers obtain necessary background to begin constructing units. Programmers begin work on background subroutines.

Personnel: Teachers (except Schmalzried) all 1/5 time. PI full time after 6/1, Schmalzried available full time after early June. Some programmers start full time in late May. Starting date: March 15, 1981.

	<u>15 March</u>	<u>April</u>	<u>May</u>	<u>June 20</u>
<u>PI</u>	order equipment;	become familiar with system		
	advertise for student programmers	select programmers		
				work with student programmers on graphics, documentation, structured approach
<u>Teachers</u>	try available CAI; background reading; select basic graphics			
			choose subject of first units	develop ideas on first units
<u>Swarthmore High School Teachers</u>	literature search	list possible trig topics and order by difficulty		
<u>Programmers</u>				begin work on graphics and I/O
<u>Consultants</u>				
Theodor H.Nelson			begin work with teachers and with programmers	
Jeffrey Travers		review cognitive literature		

FIRST SUMMER June 20-August 31, 1981

Goals: Construct first units. Re-evaluate basic approach on the basis of cognitive principles. Thereafter write, sketch out, program, and debug as many units as time permits. Construct student questionnaire and train aide. Teachers develop backlog of written units.

Personnel: All teachers full-time (high school teachers stagger vacations in July and August; PI takes his during academic year). Professor Travers 1/4 time. Programmers full time for 10 weeks (staggered). Student aide 3 days at the very end.

	20 June	July	August
All Personnel	General meeting at which Dr. J. Travers delivers report, and basic approach is discussed from cognitive perspective		General meeting during the two-day visit by Professor Bork. Basic approach discussed with him.
PI	Work with teachers and with programmers (as needed)		
Teachers	Finish writing first units	develop drafts of other units	
Programmers	finish basic graphics and I/O	code and debug first units	work on new units
Professor E. Travers	Test units as available		
	develop questionnaire		train aide
Consultants			
T. Nelson:		as needed	
J. Travers:			
report		as needed (and budget permits)	
A. Bork:			site visit

ACADEMIC YEAR Sept., 1981 - June, 1982

Goals: Further production of units; solve any graphics problems. Testing of units by student populations and by peers at other institutions.

Personnel: PI full-time (sabbatical years). Swarthmore High School teachers 1/5 time for one quarter. Student programmers and student aide part-time.

PI: Develop any needed graphics techniques. Work on the special units, such as the table of contents, index/glossary, and diagnostic unit. Work on units written during summer, and develop new units as time allows, concentrating on those which require least high school teacher input. Draft "help" program. Supervise student programmers. Send out units for peer testing at other institutions.

Student Programmers continue as before.

Student Aide helps a large sample of students from various populations through the units and then administers questionnaires. Professor E. Travers will have worked out with the various faculty the appropriate times to test the various populations.

Consultants J. Travers and T. Nelson continue as before.

FINAL PERIOD Summer, 1982: June 20 - August 31.

Goals: Reassessment of units and basic approach in the light of input from student users and peer testing at other institutions; rework as necessary. Construct "help" functions. Develop further units so as to round out project. Test units on minority populations.

Personnel: PI, high school teachers, Professor E. Travers, and student programmers as in previous summer. Student aide for two weeks testing units on minority populations.

In late June there will be an assessment of the units: the entire staff participates. Professor E. Travers presents her study of the questionnaires, the PI gives the results of peer testing at other institutions. On the basis of this assessment, there will be consideration of how to improve the units, and a final reassessment of the basic approach. There will also be a discussion of the "help" functions, based on the help requested by students from the student aide. A draft will have been prepared by the PI working with the student aide at the end of the academic year.

Based on these discussions, final plans will be drawn up for reworking existing units and creating the "help" functions. Topics for further units to round out the project will also be decided upon. It is likely that we will wish to devote a portion of the summer to working on some of the special units, such as the one for first-time users, the diagnostic unit, the "attract" mode, and the control program. Plans and schedules will be established at this time.

Having set the course for final action in late June, production will proceed as during the first summer. Only new activities are noted below.

Professor E. Travers will oversee the student aide in trying the units on minority students during August in two summer programs, Upward Bound (grades 9-12), and the College pre-enrollment program. The basic goals here will be (1) to obtain a sense of whether any modifications should be made to facilitate the use of the units by these populations, and (2) to test the "help" functions and first-time user unit.

The PI will devote some portion of August to writing the final report.

Termination date: August 31, 1982.

REFERENCES

[1] These are the impressions of our local high school teachers; presumably, this applies nationally.

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[5] Chambers, Jack A., et. al., The ABC's of CAI, 4th edition, California State University, Fresno, CA., 1979.

[6] Chambers, J. A. and Sprecher, J. W., Computer Assisted Instruction: Current Trends and Critical Issues, Communications of the ACM, June, 1980, 332-342.

[7] Creative Computing, June, 1980, p. 104.

[8] Begle, E. G., Critical Variables in Mathematics Education, Mathematical Association of America, Washington, D. C., 1979, p. 129.

[9] ibid, p. 86.

[10] ibid, p. 92.

[11] ibid, p. 65.

[12] Hickey, Albert E., Computer-Aided Instruction: A Summary of Research in Selected Areas, Entelek, Portsmouth, N. H., p. 25.

[13] Bruner, Jerome S., Toward A Theory of Instruction, Belknap Press, Cambridge, Mass., 1966.

[14] Brinkmann, E. H., Programmed Instruction as a Technique for Improving Spatial Visualization, J. Applied Psychology, 50 (1966), 179-184.

[15] Begle, op. cit., p. 89.

[16] The CONDUIT publication "Pipeline", Summer, 1978, p. 13.

[17] Chambers and Sprecher, op. cit., p. 333.

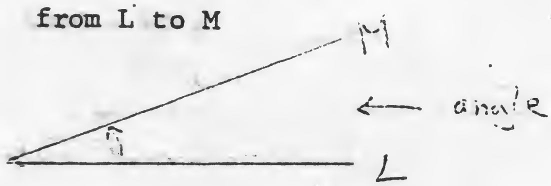
[18] Aiken, Robert M., and Braun, Ludwig, Into the 80's with Microcomputer-Based Learning, Computer, July 1980, p. 14.

APPENDIX 1: OUTLINES OF SOME POSSIBLE UNITS

1. Possible Unit on Radian Measure of Angles

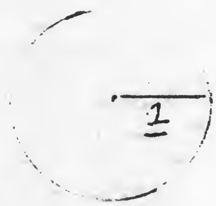
Shown on Screen / **Comments**

Given the angle from L to M



Angle chosen randomly between 3/4 and 2 radians. The "< angle" would be in a lighter color for deemphasis.

Take a unit circle.



"Unit" and "1" blink together.

exit

to continue

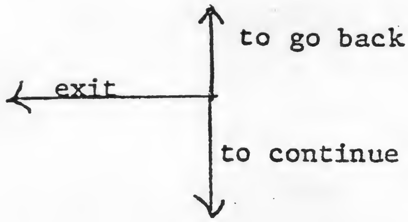
This indicates possible options available by joystick motion: push downward to go on, leftward to exit from unit (and into "menu"/table of contents)

The radian measure of the angle is obtained as follows:

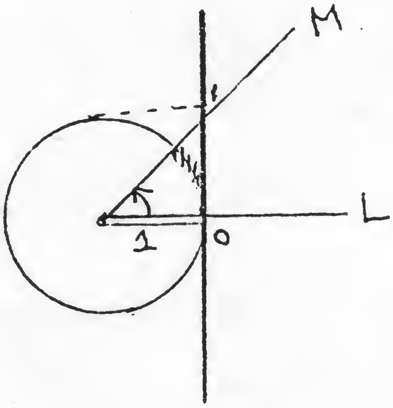
circle moves so that its center is at the vertex

the radian measure is the length of this arc

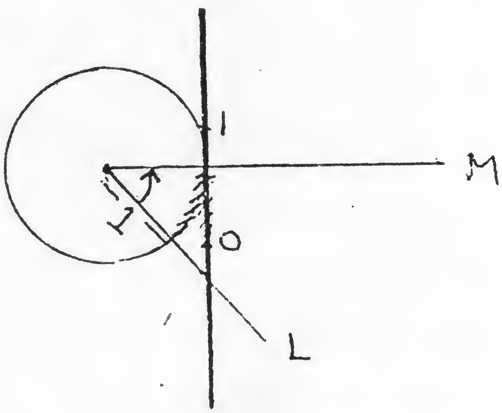
arc shown in strong color



The student readily becomes accustomed to controlling rate of presentation via joystick.



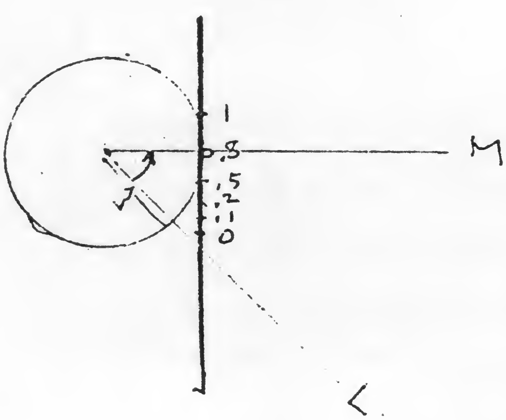
Tangent line appears to circle, numbers 0 and 1 appear on line, with color suggesting that unit line segment and radius are same length.



Circle rolls on line until the point of tangency is the end of the arc. The line segment takes on the same strong color of the arc.

In the case the radian measure is approximately <u>.8.</u>

Other points of the number scale appear on the tangent line.



Color of .8 in both places is the same (and contrasting with others) and both .8's flash.

this
way
back

exit — this way to choose your own angle to be measured

this way
to new
material

Joystick directions.

In choosing the angle, the student would be given a fixed line L, and could move M via the joystick. Of course the student could anticipate later material, but that's fine.

New material in this unit would deal with negative angles, angles greater than 360°, etc.

Quizzes for this unit would begin by giving the student an angle and a unit circle, and asking the student to use the joystick to trace the arc (in the proper direction) whose length is the radian measure of the angle. Another type of problem would give the student an angle, a unit circle with appropriate numbers on its circumference, and the ability to translate the circle and to rotate it (so that the numbers align properly), and ask for the radian measure as numerical input. Still another type of problem might give the student the tools used above (i.e., ability to translate unit circle and to roll it in either direction along a number line).

In general, students would always have the option of receiving answers (with appropriate constructions and motions). After a problem had been completed, they could choose similar problems of the same kind, harder or easier problems of the same kind, problems of a different kind, to go back over the most recent problem, or to exit from a unit.

Whenever appropriate, we would present students with estimation problems associated with a unit. In this instance, the student would simply be given an angle and asked to estimate it in radian measure. When the student does so (or calls for the answer), the answer would be given (to the degree of resolution available) along with appropriate measurement tools.

When the unit had been exhausted, the student would be given the possibility of exiting from the unit (landing at the point on the menu listing the unit), or choosing related units. For the unit described, these might include units on radian measure and π, on radian measure vs. degree measure, on what happens if non-unit circles are used, etc.

2. Rough Outline of Some Possible Polar Coordinate Units

Unit 1 Representing points in the plane by polar coordinates (Throughout, both radians and degrees would be used. For simplicity, we've only alluded to radians below.) Beginning with r positive and t between 0 and 2π, the program shows how the pair (r,t) can be used to represent a point in the plane.

The student is asked to plot some points for randomly selected pairs using the joystick; true location is shown in a different color after the student's selection has been made. The student can always elect to have the computer plot the point instead and can go further in the unit when desired. At least at first the student could call for polar coordinate "graph paper" to appear. The next stage in the unit would be to ask the student to locate (r,t), with t negative, with the same possibilities and provisions as before. Then ask the student to locate (r,t) with r negative. Explanation and comments would be provided. The student would then be asked to locate the same point twice in a row, being given different coordinate pairs. It would be observed that each point in the plane has lots of different polar coordinates, as opposed to unique cartesian coordinates. Quiz Module: The student is given randomly selected pairs (r,t) and asked to plot them; for estimation the student is given randomly selected points in the plane

and asked for their approximate polar coordinates. (After each student answer, polar coordinate graph paper is displayed and a more precise answer is given.) The student is given a polar coordinate pair (r,t) and asked for coordinates describing the same point with different t; same with different r. (N.B. in this and several other units, it might be helpful to have the keyboard "P" produce a "π".)

Unit 2 Loci of points of the form (r,t) with r=f(t). Beginning with simple examples, such as f(t)=2 or f(t)=t, the student would be asked to plot selected points (or choose to have the computer do so). The points would then be connected to give the complete curve. When the student desires, the program would move on to more complicated functions, such as f(t)=1-cos t. The student would then be allowed to input functions and try graphing them, or have them graphed.

Quiz: given functions (randomly selected from classes of graded difficulty, but having not appeared in the unit), have the student plot points and fill in the curve with the joystick. The true curve would be shown in a different color. Given the graph of a curve and various possible functions r=f(t), which could it be? which could it not be? (Reasons could be stored for the latter, and provided at appropriate times.)

There would follow a unit on changing from polar coordinates to rectangular, and vice versa. There would also be an exploratory unit, intended for browsing, which would display famous curves and their polar and cartesian forms. An option would allow points to simultaneously trace out the curve and run through an appropriate table of values, at a speed and direction controlled by the student. The student could also enter curves (with r=f(t)) for the same treatment.

INSTRUCTIONS FOR USING TRIGONOMETRY SAMPLE DISKS

E.Klotz, November 1, 1982

Use these disks on an Atari 800 with 48K memory (later versions will fit on an Atari 400 with 32K memory). Remove any game or BASIC cartridges, then boot. Begin with Disk I the first time (for the Instructions); after that, boot with any disk. Plug a joystick into Controller Jack #1. It is preferable to use a color TV, with volume set according to the user's taste.

Main points in the Instructions (on Disk I):
-move the joystick in the direction of the arrows in the lower left-hand corner of the screen. In general, pushing forward will advance to new material; pulling back will review.
-to get out of a unit, press the OPTION button.

Units have objectives, which are accessable both from the unit's title page, and at the end of the unit. Activity and Exploration units have worksheets; sample copies are enclosed for three of the units on Disk III. Units which offer multiple possibilities (such as games and explorations) have their own menus.

CONTENTS OF THE SAMPLE DISKS

DISK I: Instructions, etc., Degree Measure

Instructions
Degree Measure
Angles and Rotations
TV Calibration
Credits

DISK II: Radian Measure Excerpts

Radians: another way of measuring angles
The Radian Estimation Game
Radian Measure and Arc Length

DISK III: Sine Sampler

History of "Sine"
Sines of Angles in Right Triangles
Sines and Coordinates - Exploration*
Sines of Special Angles (Tutorial/Activity)*
The Graph of the Sine*

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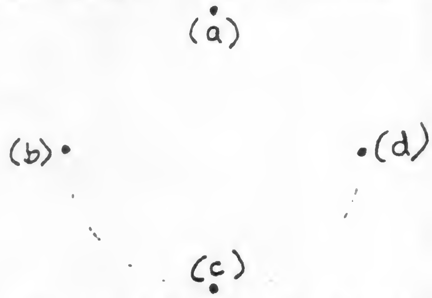
* For the last three units, see the enclosed worksheets.

Go through the entire unit at least once before attempting the worksheets. On your second time through:

1. As the unit circle is drawn, record the arc lengths at each point shown (in decimal form).



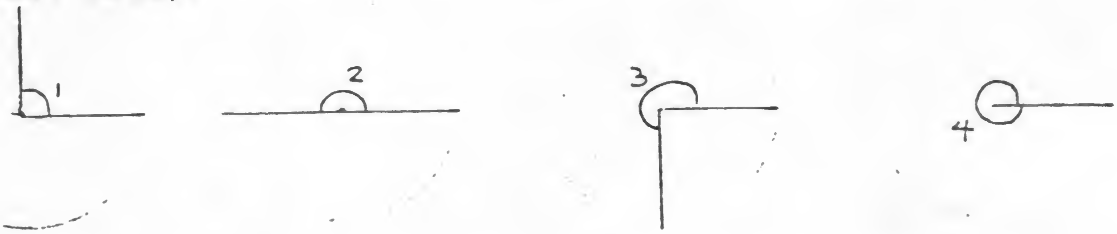
(a) ________

(b) ________

(c) ________

(d) ________

2. Now go to the sine graph (push START) and change the angle measure unit to radians (by pushing the joystick button). Write down the radian measures for the following angles (in decimal form).



(1) ________ (2) ________ (3) ________ (4) ________

3. How do the measures compare in problems 1 and 2? Why? (There is a hint in the text just before the graph is drawn)

4. 

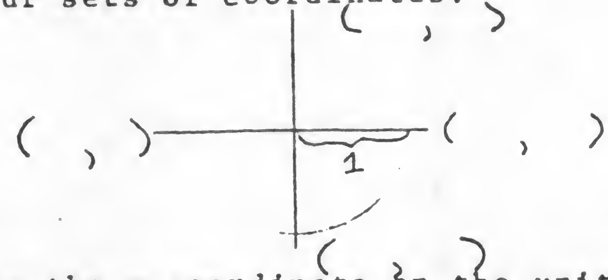
Describe in words what is happening on the screen as the unit circle is drawn.

What is the relationship between the arc on the circle and the horizontal axis?

5. On the top of the screen the arc length is given in decimal form. On the axis it is written with π. These numbers are related. How?

GRAPH OF THE SINE   WORKSHEET 2

1. Placing a unit circle on coordinate axes as shown, label the four sets of coordinates.



2. What is the y-coordinate on the unit circle when angle Θ is:

0° ________

90° ________

180° ________

270° ________

360° ________

3. Record the values of sin Θ at:

0° ________

90° ________

180° ________

270° ________

360° ________

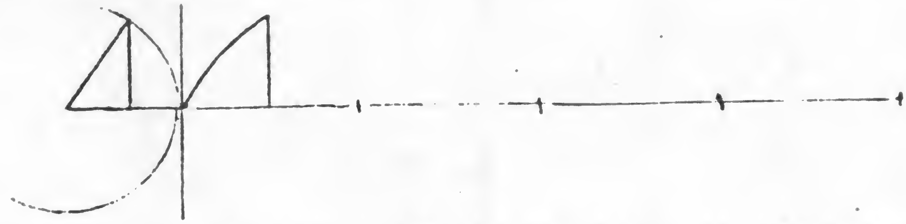
4. How do the values in problems 2 and 3 compare? Why?

5. Why does the y-coordinate on the unit circle equal the sine of the angle?

GRAPH OF THE SINE

WORKSHEET 3

The following figure appears when the sine graph is drawn:



1. What is the definition of sin Θ which is used here?

   Label all the parts of the definition in the figure.

2. What does the moving vertical line represent?

   What does the color change after π indicate?

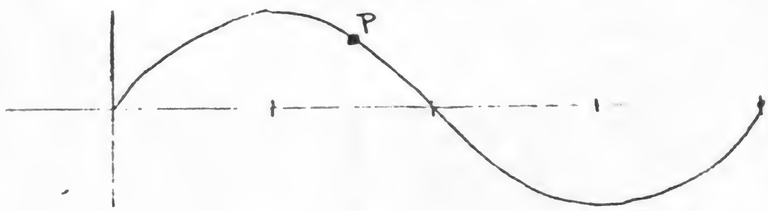
3. From the graph, tell what happens to the value of sin Θ:

   a) in the first quadrant (0°-90°) or $(0-\pi/2)$

   b) in the second quadrant (90°-180°) or $(\pi/2-\pi)$

   c) in the third quadrant (180°-270°) or $(\pi-3\pi/2)$

   d) in the fourth quadrant (270°-360°) or $(3\pi/2-2\pi)$



4. For the above point, there is a corresponding angle on the circle.

   a) Draw the angle and label it Θ.

   b) The angle measure of Θ is also represented on the horizontal axis. Locate this point and label it X.

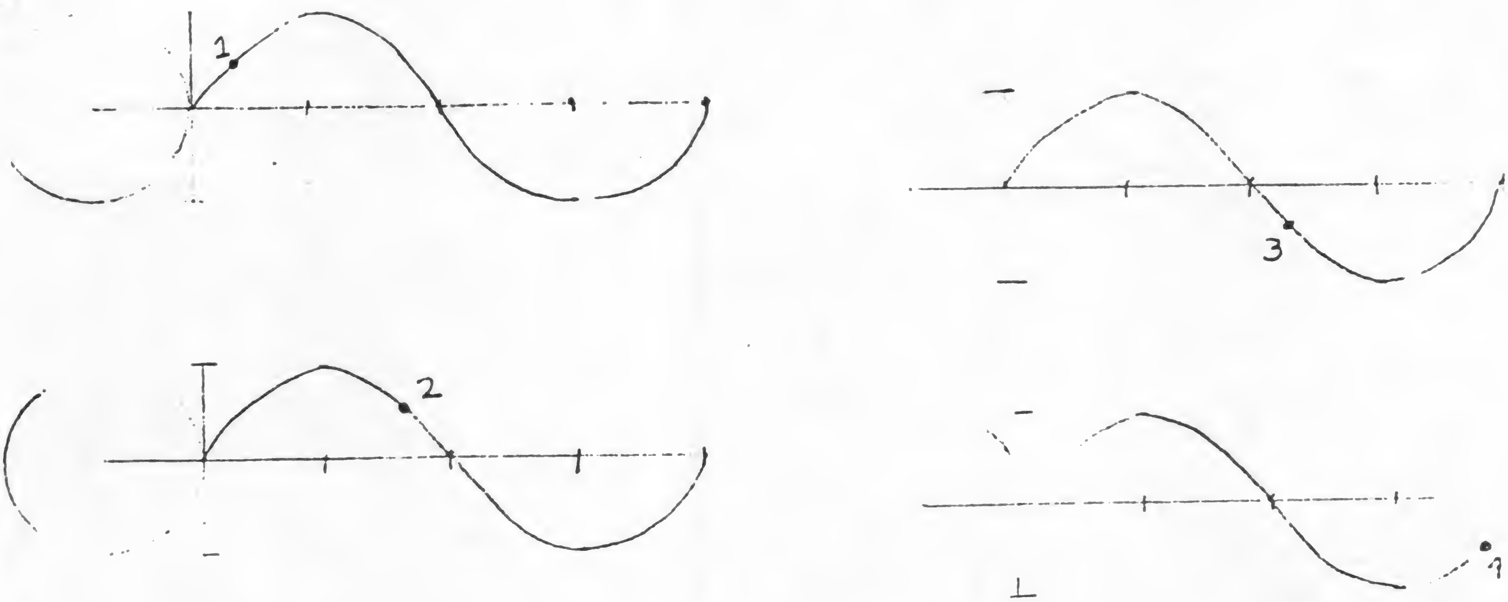
   c) Sin Θ is represented by two different vertical line segments. Draw them in.

   d) When graphing a function on coordinate axes, it is customary to have a horizontal x-axis and a vertical y-axis. In the picture, what does the horizontal axis represent?

   What does the vertical axis represent?

GRAPH OF THE SINE — WORKSHEET 4

1. Given these four points on the sine graph:



Each of these points may be thought of as originating from a point on the unit circle.
Label the corresponding points on the unit circle in each picture.
Also, indicate angle Θ by a terminal side and an arrow.

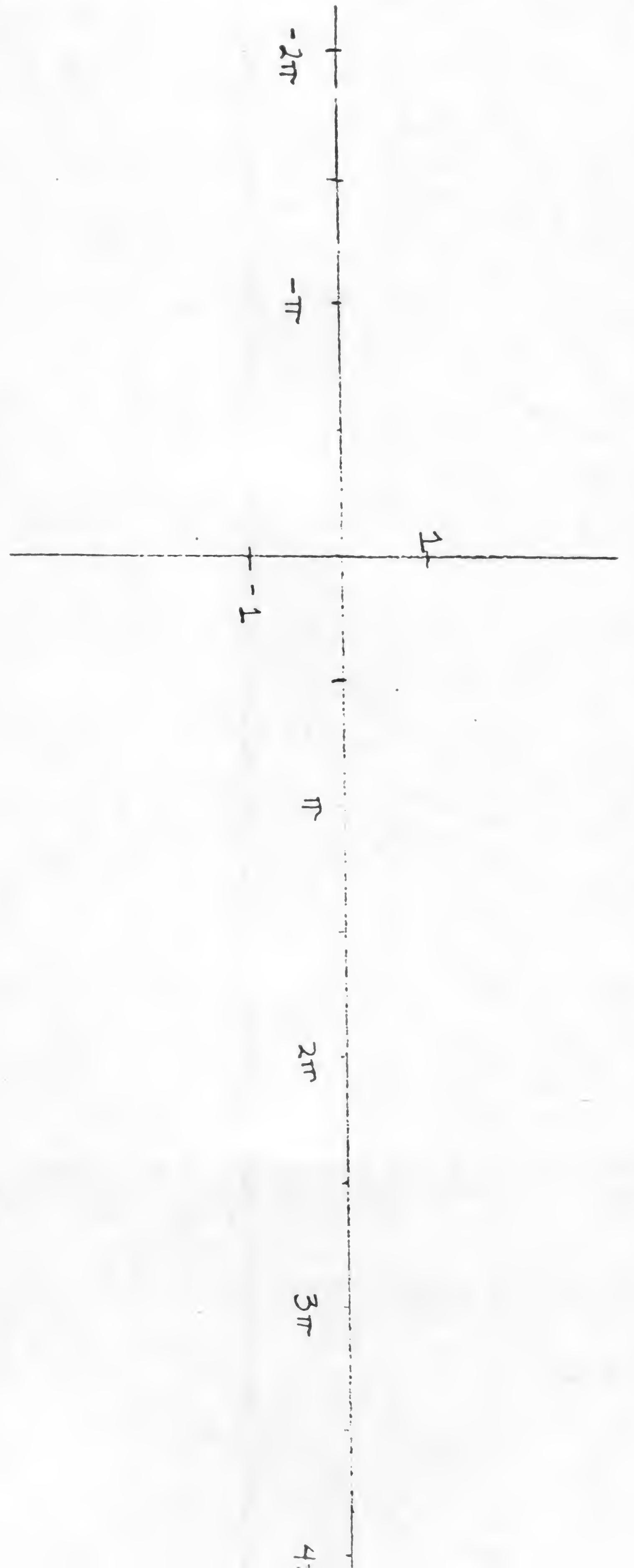
2. Use the graph provided on the next page:

First: copy what you see on the screen for the sine graph from $0 - 2\pi$.

Second: draw what you think the sine graph will look like from $2\pi - 4\pi$.

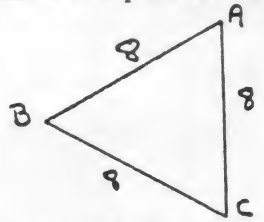
Third: draw what you think the sine graph will look like from $-2\pi - 0$.

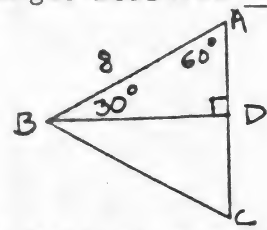
Explain the patterns you see.


-2π
-π
1
-1
π
2π
3π

SINES OF SPECIAL ANGLES (Activity) WORKSHEET 1

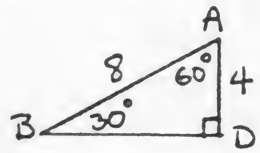
1. In an equilateral triangle, each angle measures ________°.





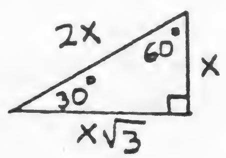
2. Think of a 30°-60°-90° triangle as half of an equilateral triangle, as shown above. Find the length of $\overline{AD}$: ____________

3.



Use the Pythagorean Theorem to find the length of $\overline{BD}$ (in simplified radical form): __________

4.



In any 30°-60°-90° triangle, the side opposite the 30° angle is equal to half the hypotenuse. Why?

5. The side opposite the 60° angle is equal to (half the hypotenuse) $\times \sqrt{3}$. Why?

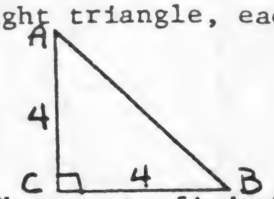
6. Use the triangle in #4 to answer the following:

$$\sin 30° = \frac{\text{opp.}}{\text{hyp.}} = \frac{x}{2x} = \_\_\_\_\_\_\_\_\_\_$$

$$\sin 60° = \frac{\text{opp.}}{\text{hyp.}} = \frac{x\sqrt{3}}{2x} = \_\_\_\_\_\_\_\_\_\_.$$

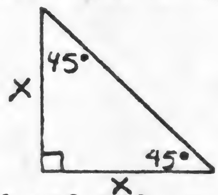
Change your last answer to a decimal, using $\sqrt{3} \approx 1.732$. ________

1. a) In an isosceles right triangle, each acute angle measures __________°.



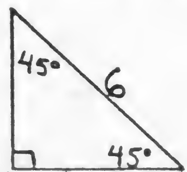
b) Use the Pythagorean Theorem to find the length of $\overline{AB}$ (in simplified form).

2. Show that in general the hypotenuse of a 45°-45°-90° triangle is equal to $\sqrt{2}$ x (the length of each leg).



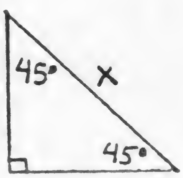
3. Suppose you know the length of the hypotenuse of a 45°-45°-90° triangle.

a) The legs must be equal in length. Why?



b) Use the Pythagorean theorem to find the length of each leg (simplified).

4. Show that in general each leg of a 45°-45°-90° triangle is equal to (half the hypotenuse) x $\sqrt{2}$.



5. Use the triangle in #4 to complete the following:

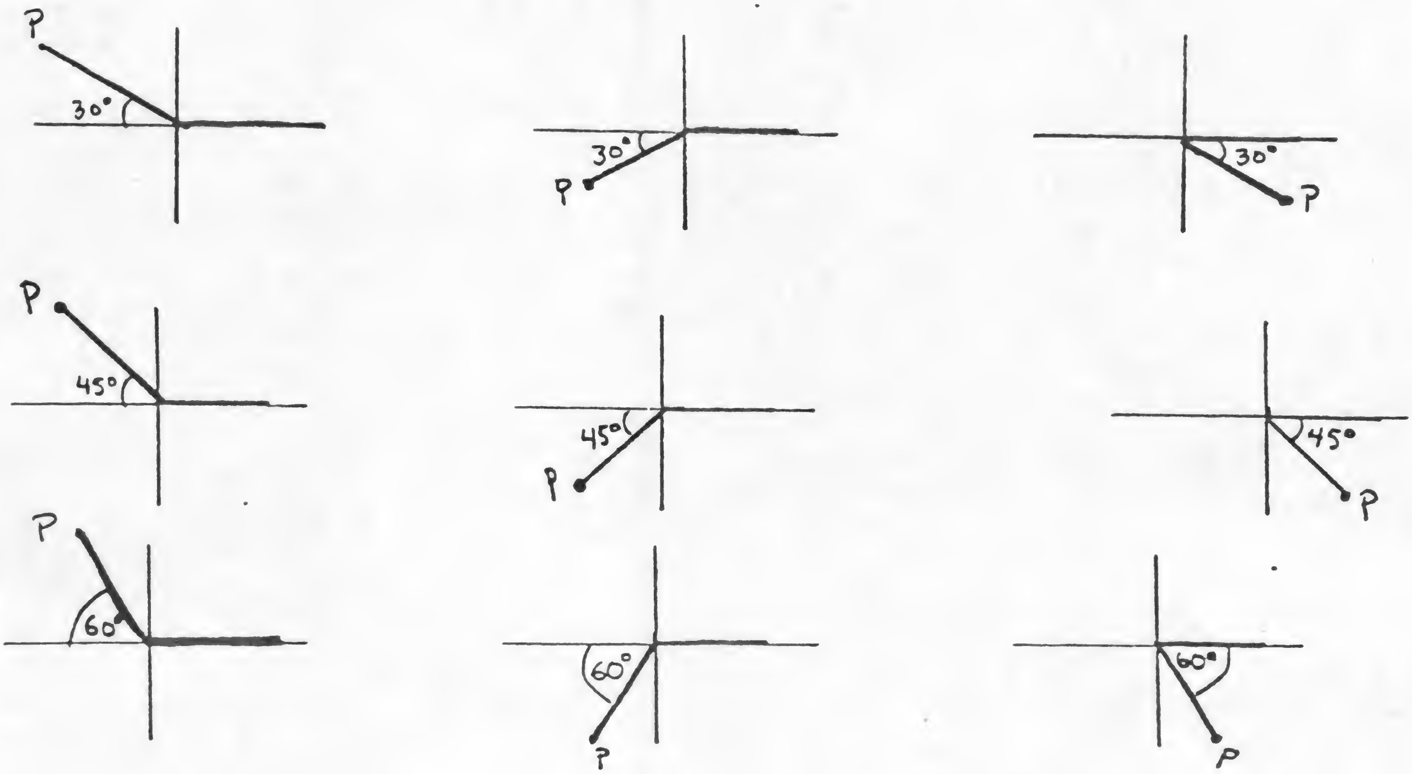
$\sin 45° = \dfrac{\text{opp.}}{\text{hyp.}} = \dfrac{\frac{x}{2}\sqrt{2}}{x} =$ __________.

Change your answer to decimal form, using $\sqrt{2} \approx 1.414$: __________

SINES OF SPECIAL ANGLES WORKSHEET 3

1. Match each of the special angles with its picture below:

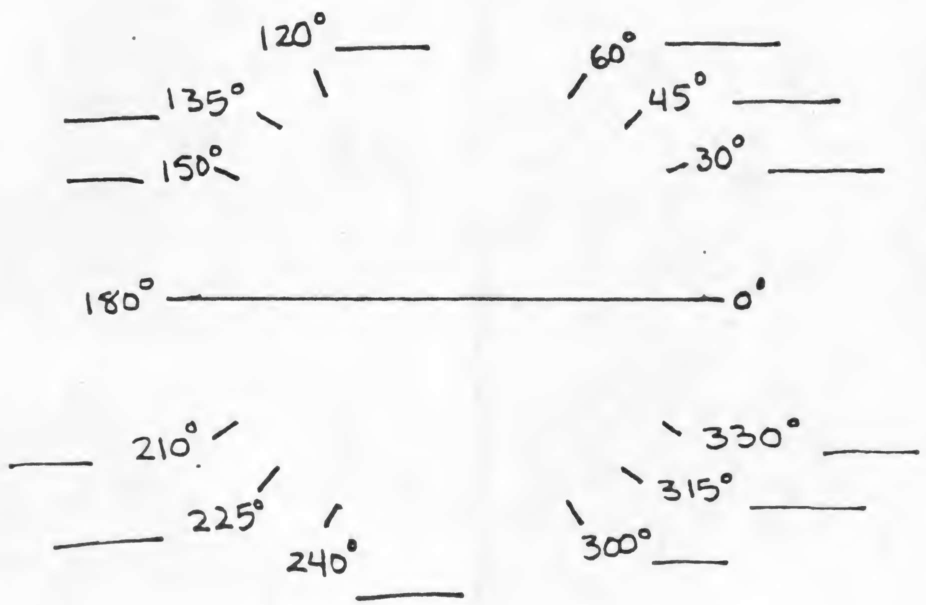
120°	210°	300°
135°	225°	315°
150°	240°	330°



2. For which of the angles above is the y-coordinate negative?

SINES OF SPECIAL ANGLES WORKSHEET 4

Fill in the sine (in radical form) for each of the angles in the space provided. Look for patterns, It may not be necessary to look back at the activity for every angle, but feel free to check your answers.



QUESTIONS:

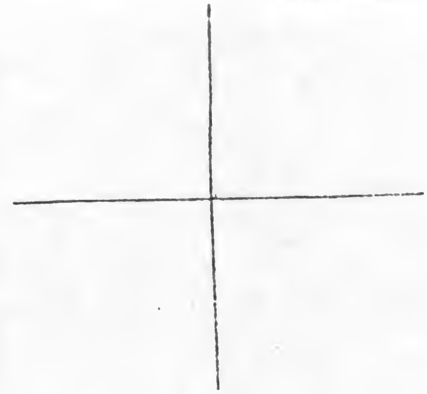
1. For which of these angles is the sine positive?
   Why?

2. For which of these angles is the sine negative?
   Why?

3. For which angles is the sine either $\frac{1}{2}$ or $\frac{-1}{2}$?
   All of these make an angle of ______° with the horizontal axis.
4. For which angles is the sine either $\frac{\sqrt{2}}{2}$ or $\frac{-\sqrt{2}}{2}$?
   All of these make an angle of _____° with the horizontal axis.
5. For which angles is the sine either $\frac{\sqrt{3}}{2}$ or $\frac{-\sqrt{3}}{2}$?
   These make and angle of _______° with the horizontal axis.

SINES AND COORDINATES (Exploration)

WORKSHEET 1



1. As a brief review of the Cartesian coordinate system:
   a) Label the x and y axes.
   b) Label the quadrants I-IV.
   c) For each quadrant, tell whether the x and y values are positive or negative.

2. Try the activity several times. Make sure that you try at least one point in each quadrant.
   In which quadrants is $\sin\theta$ negative?
   Explain why.

3. Become familiar with the quadrantal angles, constructed as follows:

   a) Try the point (6,0).
   Fill in the following: X = 6 Y = 0 R = _______

   θ = _______ SINθ = _________

   b) Choose any point which will make a 90^{0} angle, and fill in the following: X = _____ Y = _______ R = _______

   $\theta = 90^{0}$ SINθ = ___________

   c) Choose any point which will make a 180^{0} angle, and fill in the following: X = _____ Y = _______ R = _________

   $\theta = 180^{0}$ SIN θ = ____________

   d) Choose a point which will make a 270^{0} angle.
   Fill in: X = ______ Y = _______ R = _________

   $\theta = 270^{0}$ SINθ = ________________

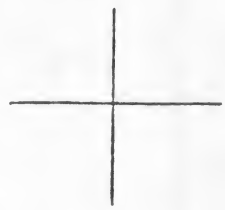
   e) Try the point (0,0). What happens?

SINES AND COORDINATES (Exploration) WORKSHEET 2

1. Enter the point (-5,12), and press the red button.

   a) Draw what you see on the screen.

X = Y = R =



[θ = sinθ =]

   b) Label R on your drawing. How was the value of R calculated?

   c) How was sin θ calculated?

   (Note: in most cases, you would not be expected to calculate the measure of θ, unless you had sine tables available.)

2. For some points, the value of R is an integer.
   Find the value of R for the following points:

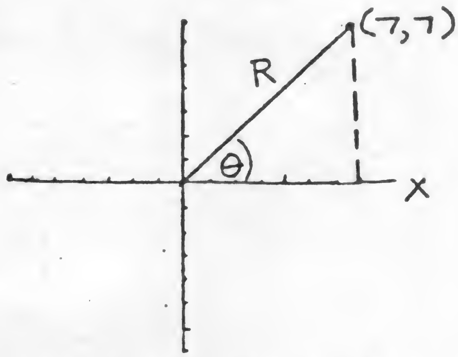
   (3,4): R = ________ (-4,-3): R = __________

   (8,-6): R = ________ (-9,12): R = __________

   (-5,-12): R = ________

   (You may check your answers on the computer. Note that R is always positive.)

1.



a) What is the measure of θ here? Explain your reasoning.

b) Find the value of R in simplest radical form:

c) Find the value of $\sin\theta$ in simplest radical form:

d) Enter the point (7,7) in the computer activity, and fill in the decimal approximations which are given.

R = __________ $\sin\theta$ = ____________

2. Try the point (10,10) and fill in the following:

R = ___________ θ = _________ $\sin\theta$ = _________

Notice that θ and $\sin\theta$ are the same as in #1, even though the value of R has changed.

3. Special angles which are multiples of 45° can be easily constructed. For each angle below, give a point which will make that angle. Then find the values of R and $\sin\theta$.
The first one has been done, as an example.

a) 45°: (7,7) R = 9.9 $\sin\theta$ = .707

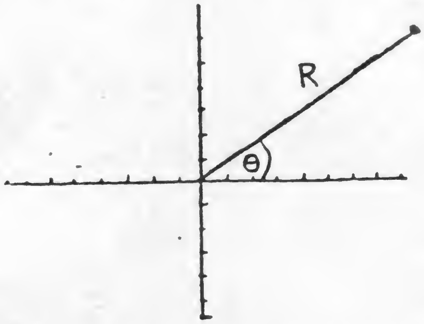
b) 135°:

c) 225°:

d) 315°:

SINES AND COORDINATES (Exploration) WORKSHEET 4

1. Given the point (8,6):

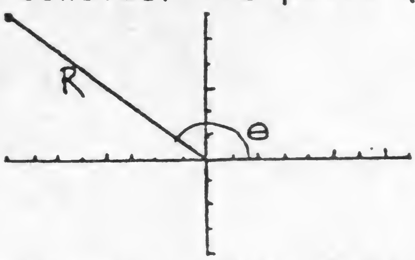


a) Calculate the value of R: ____________

b) Calculate $\sin\theta$: ______________

c) Enter this point in the computer activity. Check your results, and find the value of θ : ___________

2. Consider the point (-8,6):

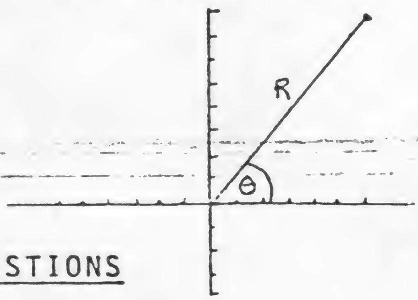


a) Calculate R: ________________

b) Calculate $\sin\theta$: ____________

c) Enter the point, and find θ : _________

3. Try the point (6,8):



a) Calculate R: __________________

b) Calculate $\sin\theta$: _________

c) Find θ : ____________

QUESTIONS

4. In # 1-3, why are all the R values equal?

5. Why are the sines equal in #1 and #2?

6. How are the angle measures in #1 and #2 related?

7. How are the angle measures in #1 and #3 related?